Abstracts of the talks of the Paris-London Analysis Seminar

Session 63, March 20, 2026 in London (Imperial College)

Scott Armstrong (CNRS and Sorbonne Université)
Anomalous diffusivity and regularity for random incompressible flows

Abstract. We consider the behavior of Brownian motion in a ”turbulent” drift, that is, a stationary, incompressible random drift field with slowly decaying correlations. In this setting, one expects the variance of the displacement to grow faster than linearly in time, with an exponent determined by the correlation structure of the drift. This behavior was predicted by physicists in 1990 using perturbative renormalization group heuristics. We formulate the problem as a PDE problem via the associated divergence-form drift–diffusion operator, which has self-similar (or multifractal) coefficients. We apply a scale-by-scale coarse-graining scheme to this operator. At each scale, this produces an effective Laplacian whose diffusivity depends on the scale, together with quantitative control of the approximation error. In other words, we use methods originating in quantitative homogenization theory, but we must iteratively perform infinitely many homogenizations, and the operator never “finishes” homogenizing because of its self-similar structure. This may be viewed as a rigorous version of the perturbative RG heuristics. A crucial role is played by anomalous regularization, that is, regularity estimates for solutions that are independent of the bare molecular diffusivity. The work I will describe is based on our joint paper with A. Bou-Rabee and T. Kuusi; the link is here: https://arxiv.org/abs/2601.22142

Giada Franz (CNRS and Université Gustave Eiffel)
Morse theory for minimal surfaces: emanating flow lines

Abstract. Morse theory provides a framework for studying the infinite-dimensional space of embedded hypersurfaces in a fixed ambient manifold through the analysis of the area functional. A special role in this picture is played by minimal hypersurfaces, which are precisely the critical points of this functional. By analogy with classical Morse theory, one expects that a minimal hypersurface of Morse index I should admit an I-dimensional unstable manifold emanating from it. In this talk, I will present recent results that make this heuristic precise by constructing and characterizing ancient solutions to mean curvature flow that emanate from minimal hypersurfaces. Particular focus will be given to the case of hypersurfaces with boundary.

Rita Teixeira da Costa (University of Cambridge)
On the stability of rotating black holes

Abstract. I will discuss some recent work with collaborators on the stability of rotating black hole solutions to the Einstein vacuum equations.

Nikolaos Zygouras (University of Warwick)
From subcritical to critical Singular SPDEs

Abstract. The past decade has witnessed a revolution in the treatment of singular stochastic partial differential equations (SPDEs) through mainly Hairer's Theory of Regularity Structures, Gubinelli-Imkeller-Perkowski Paracontrolled Distributions and also via renormalisation approaches. These theories, however, are limited by the so-called "criticality" or "critical dimension". The first steps towards criticality are now shyly being made. After giving a general taste of singular SPDEs, I will present some of the first steps at criticality and discuss in particular the Critical 2d Stochastic Heat Flow, which was constructed jointly with Francesco Caravenna and Rongfeng Sun, as a non-trivial solution of the Stochastic Heat Equation in dimension 2.

Session 62, December 12, 2025 in Paris (Institut Henri-Poincaré)

Yann Brenier (CNRS et Université Paris Saclay)
Solving initial value problems by space-time convex optimization

Abstract. I will discuss a possible strategy to solve the Cauchy problem for nonlinear evolution PDEs by space-time convex optimization based on their weak formulation. One of the simplest example is the quadratic porous medium equation for which the Aronson-Benilan inequality is sharply used to prove that the strategy works for arbitrarily long time intervals. A similar result holds true for the Burgers equation. For the more challenging Euler equations, the concept of subsolution (in the sense of convex integration theory) plays a crucial role. Finally, I will mention how the Einstein equations in vacuum can be considered in that framework.

Adina Ciomaga (Université Paris Cité, Romanian Academy Iaşi Branch)

Nonlocal Hamilton Jacobi equations

Abstract. Hamilton Jacobi equations with nonlocal terms of integro - differential type naturally arise in optimal control problems, front propagation, and stochastic processes with jumps. These equations typically take the form

$$u - I[u](x) + H(x, Du) = 0, \text{ in } \mathbb{R}^d.$$

where I is a nonlocal operator associated with a Lévy measure and encodes long-range interactions or jump dynamics and H is a Hamiltonian. Firstly, I will discuss the analytical framework for viscosity solutions in the presence of nonlocal operators, emphasizing well-posedness, comparison principles, and stability under natural structural conditions on the Hamiltonian H and the nonlocal operator I. Secondly, I will address regularity of solutions for Hamilton–Jacobi equations with integro-differential terms. This is a delicate issue due to the interplay between the first-order nonlinear Hamiltonian and the nonlocal operator. Unlike the purely local case, where the coercivity of H or convexity assumptions can yield Lipschitz continuity of solutions, the presence of a nonlocal operator I introduces long-range interactions that may either enhance or obstruct regularization effects. In many cases, the nonlocal term provides a compensating effect that prevents singularity formation and yields regularity of solutions. However, optimal regularity remains mostly open and strongly depends on fine properties of the jump measure and the interaction between nonlocal term and the nonlinear Hamiltonian.

Leonid Parnovski (University College London)

Bethe-Sommerfeld Property of multi-dimensional Schrödinger operators with periodic and almost-periodic potentials

Abstract. This will be a survey talk about the Bethe-Sommerfeld property (spectrum containing a semi-axis) of periodic and almost-periodic operators. I will describe known results as well as open problems. This talk will be not very technical and will contain no proofs.

Oleg Zaboronski (University of Warwick)
Asymptotic expansions for a class of Fredholm Pfaffians and interacting particle systems

Abstract. Motivated by the phenomenon of duality for interacting particle systems we introduce two classes of Pfaffian kernels describing a number of Pfaffian point processes in the 'bulk' and at the 'edge'. Using the probabilistic method due to Mark Kac, we prove two Szegő-type asymptotic expansion theorems for the corresponding Fredholm Pfaffians.
The idea of the proof is to introduce an effective random walk with transition density determined by the Pfaffian kernel, express the logarithm of the Fredholm Pfaffian through expectations with respect to the random walk, and analyse the expectations using general results on random walks. We demonstrate the utility of the theorems by calculating asymptotics for the empty interval and non-crossing probabilities for a number of examples of Pfaffian point processes: coalescing/annihilating Brownian motions, massive coalescing Brownian motions, real zeros of Gaussian power series and Kac polynomials, and real eigenvalues for the real Ginibre ensemble. (Joint work with Will FitzGerald and Roger Tribe.)

Session 61, November 7, 2025 in London (King's College London)

Claudia Garetto (Queen Mary University of London)
Schrödinger type equations with singular coefficients

Abstract. In this talk I will present some recent results obtained in collaboration with Alexandre Arias Junior (São Paulo), Alessia Ascanelli (Ferrara) and Marco Cappiello (Torino) on the well-posedness of the Cauchy problem for Schrödinger type equations with singular coefficients. Different function spaces will be used as well as regularisation methods leading to nets of

smooth solutions.

Frédéric Klopp (Sorbonne Université)

The discrete Mathieu operator at non real coupling in the adiabatic limit

Abstract. We study the asymptotics of the spectrum of the discrete Mathieu operator at non real coupling in the adiabatic limit. More precisely, for $(\lambda, p_0, \zeta_0) \in \mathbb{C} \setminus \mathbb{R} \times \mathbb{C}^2$, on $[\![1, N]\!]$, we consider the operator

$$(H^N_{p_0,\zeta_0,\lambda} u)(n) := \frac{1}{2}\left(e^{ip_0}u(n+1) + e^{-ip_0}u(n-1)\right) + \lambda \cos\left(\frac{2\pi n}{N} + \zeta_0\right) u(n),$$

with periodic boundary conditions and compute its spectrum in the adiabatic limit, that is, for N large. When $\lambda = i$ and $\zeta_0 = p_0 = 0$, the spectrum was conjectured to be the Scottish flag (see first figure below).
Using a complex WKB method for finite difference equations, for N large, we derive an asymptotic quantization condition for the eigenvalues with "explicit" coefficients. The analysis of this quantization condition shows that, up to errors of order N^{-2}, the spectrum of $H^N_{p_0,\zeta_0,\lambda}$ lies on finitely many real analytic curves that depend on (λ, p_0, ζ_0) and that are described "explicitly".
This work is joint with I. Oltman (Northwestern Univ.).

Marco Marletta (University of Cardiff)

Pencils numerical ranges, resolvent estimates and pseudospectra

Abstract. This will be a talk of two parts. In the first part, starting from work with Sabine Bögli, I shall describe how *pencil* numerical ranges may be used to improve both localisation estimates for operator spectra, and resolvent estimates. This will be applied to examples including non-selfadjoint Dirac and Maxwell systems (joint work with Bögli, Ferraresso and Tretter). In the second part, which is recent joint work with Lyonell Boulton, I shall describe two-sided resolvent estimates for a problem first proposed to us by Brian Davies. The methods for this second part are different, rather problem-specific, and depend upon transformator kernel methods and a Carleman-type inequality of Oscar Bandtlow.

Eric Séré (Université Paris Dauphine)

The lowest eigenvalue of Dirac-Coulomb operators: results and open problems

Abstract. This talk is based on joint works with J. Dolbeault, M.J. Esteban and M. Lewin. Consider an electron in the attractive Coulomb potential generated by a positive finite measure representing an external charge density. If the total charge is fixed, it is well known that the lowest eigenvalue of the corresponding Schrödinger operator is minimized when the measure is a delta: this is easily proved thanks the classical Rayleigh quotient minimisation principle. We investigate the conjecture that the same holds for the relativistic Dirac-Coulomb operator: in that case the lowest eigenvalue lies in a spectral gap and the question is much more difficult. We give conditions ensuring that the Dirac-Coulomb operator has a natural self-adjoint realisation and that its eigenvalues are given by min-max formulas. Then we define a critical charge such that, if the total charge is fixed below it, then there exists a measure minimising the first eigenvalue of the Dirac-Coulomb operator. We find that this optimal measure concentrates on a compact set of Lebesgue measure zero.

Session 60, June 20, 2025, in Paris (Institut Henri-Poincaré)

Tobias Barker (University of Bath)

Critical norm blow-up rates for the energy supercritical nonlinear heat equation

Abstract. In joint work with Jin Takahashi (Institute of Science Tokyo) and Hideyuki Miura (Institute of Science Tokyo), we study the behavior of the scaling critical Lebesgue norm for blow-up solutions to the nonlinear heat equation (the Fujita equation). For the energy supercritical nonlinearity, we give estimates of the blow-up rate for the critical norm. Time permitting, upcoming work for the Navier-Stokes equations may also be discussed.

Yannick Guedes Bonthonneau (CNRS, Université Sorbonne Paris Nord)

Tunneling formula for the planar magnetic laplacian.

Abstract. In a joint work with Søren Fournais, Léo Morin and Nicolas Raymond, we obtain an asymptotic formula for the tunneling between magnetic wells. More precisely, in the presence of two symmetries, under some technical assumptions, if the infimum of the magnetic field is realized by two

non-degenerate wells, we show that the bottom of the spectrum of the corresponding magnetic laplacian is comprised of two eigenvalues, separated from the rest of the spectrum by a gap of size $\sim h^2$, while their difference, up to a polynomial factor is equivalent to a decaying exponential.
Previous results included exponentially small upper bounds on said difference, and a similar equivalent in the (even more) rigid case of radially symmetric wells. Our result comes as an answer to the works of Helffer and Morame of the nineties.
I will in particular discuss a key assumptions on the geometry of the level sets of the magnetic field in the complex domain.

Andreia Chapouto (CNRS, Université de Versailles Saint-Quentin)
Deep- and shallow-water limits of statistical equilibria for the intermediate long wave equation

Abstract. he intermediate long wave equation (ILW) models the internal wave propagation of the interface in a stratified fluid of finite depth, providing a natural connection between the deep-water regime (= the BO regime) and the shallow-water regime (= the KdV regime). Exploiting the complete integrability of ILW, I will discuss the statistical convergence of ILW to both BO and KdV, namely the convergence of the higher order conservation laws for ILW and their associated invariant measures. In particular, as KdV possesses only half as many conservation laws as ILW and BO, we observe a novel 2-to-1 collapse of ILW conservation laws to those of KdV, which yields alternative modes of convergence for the associated measures in the shallow-water regime. This talk is based on joint work with Guopeng Li and Tadahiro Oh.

Grigorios Pavliotis (Imperial College London)
Linearization of ergodic McKean SDEs and applications

Abstract. We study the mean field McKean SDE and the corresponding McKean-Vlasov PDE. We start by showing that, under the assumptions on the confining and interaction potentials that ensure uniform propagation of chaos, the nonlinear, nonlocal McKean-Vlasov PDE is exponentially close, in time, to a linear, local Fokker-Planck PDE. We then show how this linearization scheme can be used to provide simple, alternative proofs to the convergence of the maximum likelihood estimator and to the diffusive approximation for the mean field SDE. Finally, we discuss about the applicability of this linearization approach to the case where the mean field PDE exhibits phase transitions, i.e. non-uniqueness of stationary states.

Session 59, March 27, 2025, in London (University College London)

Louis Ioos (CY Cergy University)

Looking for canonical Kähler metrics through balanced projective embeddings

Abstract. The search for canonical Kähler metrics on projective manifolds is an attempt to extend the uniformization theorem for Riemann surfaces to general dimensions. This search has made significant progresses in the last decades, culminating in what is now called the Yau-Tian-Donaldson program. In this talk, I will explain the role played in this program by the notion of balanced projective embeddings, first introduced by Bourguignon, Li and Yau, and show how they apply to the study of Kähler-Ricci solitons.

Laura Monk (University of Bristol)

Typical hyperbolic surfaces have an optimal spectral gap

Abstract. The first non-zero Laplace eigenvalue of a hyperbolic surface, or its spectral gap, measures how well-connected the surface is: surfaces with a large spectral gap are hard to cut in pieces, have a small diameter and fast mixing times. For large hyperbolic surfaces (of large area or large genus g, equivalently), we know that the spectral gap is asymptotically bounded above by 1/4. The aim of these talks is to present joint work with Nalini Anantharaman, where we prove that most hyperbolic surfaces have a near-optimal spectral gap. That is to say, we prove that, for any $\varepsilon > 0$, the Weil-Petersson probability for a hyperbolic surface of genus g to have a spectral gap greater than $1/4 - \varepsilon$ goes to one as g goes to infinity. This statement is analogous to Alon's 1986 conjecture for regular graphs, proven by Friedman in 2003. I will present our approach, which shares many similarities with Friedman's work, and introduce new tools and ideas that we have developed in order to tackle this problem.

Stéphane Nonnemacher (Université Paris-Saclay, Orsay)

Random eigenstates of the Quantum Cat Map

Abstract. Long standing conjectures in Quantum Chaos concerns the equidistribution and statistical properties of eigenstates of quantized chaotic sys-

tems, in the semiclassical/small wavelength regime. On the macroscopic scale, one expects Quantum Unique Ergodicity: all eigenmodes should asymptotically equidistribute across the classically allowed phase space. At the microscopic (or wavelength) scale, the eigenmodes are expected to enjoy the same statistical properties as monochromatic random waves (Berry's random wave conjecture).
So far, results on Berry's conjecture have been obtained for random quasimodes of chaotic systems (e.g. the Laplacian on a closed manifold of negative curvature), localized in certain spectral windows. The reason to consider quasimodes is due to the fact that for these systems, the eigenvalues are expected to be nondegenerate, preventing any freedom in the definition of eigenmodes.
The quantized hyperbolic automorphisms of the 2-torus, also known as "Quantum Cat Maps", form a toy model of quantized chaotic systems, which enjoys atypical properties. In particular, in the semiclassical limit, the model can enjoy "maximally large" spectral multiplicities (in the context of hyperbolic surfaces, such multiplicities would saturate Bérard's bound for the remainder in Weyl's law).
These large multiplicities allow us to consider random eigenbases of the Quantum Cat Map. We prove that, with high probability, those random eigenbases are equidistributed at macroscopic and down to mesoscopic (algebraically small) scales. We also show that the local statistical properties of these random eigenstates converge to those of standard Gaussian random states, which, for this toy model, is the analogue of Berry's random wave model.
This is joint work with Nir Schwartz.

Sebastian van Strien (Imperial College London)

The Thurston algorithm for real entire transcendental post-singularly finite maps

Abstract. Thurston's characterisation theorem gives a necessary and sufficient condition for when a branched covering map of the sphere (for which the orbits of the branch points have finite cardinality) can be realised by a rational map. In spite of progress, an analogous result for entire maps of the complex plane is not yet known. In this talk, I will discuss a somewhat different approach in the setting of real entire maps whose post-singular set is real and has finite cardinality.

Session 58, December 13, 2024 in Paris (Institut Henri-Poincaré)

Nathalie Ayi (Sorbonne Université)

Large Population Limits for Interacting Particle Systems on Weighted Graphs

Abstract. When studying interacting particle systems, two distinct categories emerge: indistinguishable systems, where particle identity does not influence system dynamics, and non-exchangeable systems, where particle identity plays a significant role. One way to conceptualize these second systems is to see them as particle systems on weighted graphs. In this talk, we focus on the latter category. Recent developments in graph theory have raised renewed interest in understanding large population limits in these systems. Two main approaches have emerged: graph limits and mean-field limits. While mean-field limits were traditionally introduced for indistinguishable particles, they have been extended to the case of non-exchangeable particles recently. In this presentation, we introduce several models, mainly from the field of opinion dynamics, for which rigorous convergence results as N tends to infinity have been obtained. We also clarify the connection between the graph limit approach and the mean-field limit one. The works discussed draw from several papers, some co-authored with Nastassia Pouradier Duteil and David Poyato.

Nicolas Burq (Université Paris-Saclay, Orsay)

Nonlinear interpolation and the flow map of quasilinear equations

Abstract. I will present an abstract result showing that for the flow map of a quasilinear problem, both the continuity of the flow as a function of time and the continuity of the data-to-solution map follow automatically from the estimates that are usually proven when establishing the existence of solutions: propagation of regularity via tame a priori estimates for higher regularities and contraction for weaker norms. This result is actually the consequence of an interpolation theorem for nonlinear functionals defined on scales of Banach spaces that generalize Besov spaces. Our analysis is self-contained and independent of any previous results about interpolation theory. It depends solely on the concepts of Friedrichs' mollifiers, seen through the formalism introduced by Hamilton, combined with the frequency envelopes introduced by Tao and used recently by two of the authors and others to study the Cauchy problem for various quasilinear evolutions in partial differential equations. Though I will not present the abstract general result, I will explain its proof and illustrate on some examples how we can

easily check the assumptions.
This is a joint work with T. Alazard, M. Ifrim, D. Tataru and C. Zuily.

Mikhail Karpukhin (University College London)
Eigenvalues and minimal surfaces

Abstract. Given a Riemannian surface, the study of sharp upper bounds for Laplacian eigenvalues under the area constraint is a classical problem of spectral geometry going back to J. Hersch, P. Li, S.-T. Yau and N. Nadirashvili. The particular interest in this problem stems from the remarkable fact that the optimal metrics for such bounds arise as metrics on minimal surfaces in spheres. In the talk I will survey recent results on the subject with an emphasis on the fruitful interaction between the geometry and spectral bounds. In particular, I will describe a surprisingly effective method of constructing new minimal surfaces based on the eigenvalue optimisation with a prescribed symmetry group.

Stephen Lynch (King's College London)
Singularities in mean curvature flow

Abstract. Mean curvature flow moves a hypersurface in Euclidean space with velocity equal to its mean curvature vector. This evolution is described by a nonlinear weakly parabolic system. Variationally, it is a formal gradient flow for the volume functional. Solutions to mean curvature flow exhibit a huge variety of different kinds of singularities. For solutions which move monotonically (have nowhere vanishing mean curvature), however, these singularities exhibit enough structure so that they might eventually be completely classified. We will discuss the now essentially complete picture for surfaces in $\mathbb{R}^3$ developed over the last 40 years, and then explore the dramatically more complicated setting of 3-dimensional hypersurfaces in $\mathbb{R}^4$.

Session 57, October 18, 2024 in London (Queen Mary University of London)

Oana Ivanovici (CNRS and Sorbonne Université)
Dispersive estimates for the wave equation outside an ellipsoid

Abstract. We consider the wave equation with Dirichlet boundary condition outside an ellipsoid in $\mathbb{R}^3$ and obtain sharp dispersive bounds for the linear flow.

Jacek Jendrej (CNRS and Université Sorbonne Paris Nord)
Recent progress on the problem of soliton resolution

Abstract. Dispersive partial differential equations are evolution equations (that is, involving the time variable) whose solutions preserve the energy, but can still decay in large time due to the fact that various frequencies propagate with distinct velocities. In some cases, there exist special solutions called solitons, which do not change their shape as time passes. The Soliton Resolution Conjecture predicts that, apart from exceptional cases, solitons are the only obstruction to the decay of solutions. More precisely, every solution eventually decomposes into a superposition of solitons and a decaying term called radiation. We will discuss the conjecture in the context of the critical wave maps equation, which is the analogue of the wave equation for maps from $\mathbb{R}^2$ to $\mathbb{S}^2$. The solitons correspond to harmonic maps, which were classified by Eells and Wood in 1976. We consider equivariant solutions, which are solutions having a specific symmetry preserved by the flow. In a joint work with Andrew Lawrie, we prove that soliton resolution holds for these solutions. Our proof hinges on an analysis of collisions of solitons and an appropriate localized Lyapunov functional, which together allow to prove a no return lemma for multisoliton configurations. Building on some of these ideas, we solve, in a joint work with Andrew Lawrie and Wilhelm Schlag, an analogous problem for the harmonic map heat flow of Eells and Sampson (1964) without assuming any symmetry of the initial data.

Monica Musso (University of Bath))
Delaunay-like compact equilibria in the liquid drop model

Abstract. The liquid drop model was originally introduce by Gamow in 1928 to model atomic nuclei. The model describes the competition between surface tension (which keeps the nuclei together) and Coulomb force (which corresponds to repulsion among the protons). Equilibrium shapes correspond to sets in the 3-dimensional Euclidean space which satisfies an equation that links the mean curvature of the boundary of the set to the Newtonian potential of the set. In this talk I will present the construction of toroidal surfaces, modelled on a family ofDelaunaysurfaces, with large volume which provide new equilibrium shapes for the liquid drop model. This work is in collaboration with M. del Pino and A. Zuniga.

Cagri Sert (University of Warwick)

Projections and sumsets of self-affine fractals

Abstract. I will explain some results from our ongoing work with Ian D. Morris which aims at a systematic study of projections of self-affine fractals. These include

- extending Falconer's classical results on the Hausdorff dimension of self-affine fractals to their projections;
- a mechanism to construct self-affine fractals in dimensions at least four, whose exceptional projections the sense of Marstand Projection Theorem contain higher degree algebraic varieties in Grassmannians (such constructions are not possible in lower dimensions);
- applications including constructing self-affine fractals whose sumsets have lower than expected dimension without satisfying a usual form of resonance.

Session 56, June 21, 2024 in Paris (Institut Henri-Poincaré)

Frédéric Charve (Université Paris-Est Créteil)

Hidden asymptotics for the weak solutions of the strongly stratified Boussinesq system without rotation

Abstract. It is known that when the Froude number goes to zero, the solutions of the strongly stratified Boussinesq system tend towards those of a 3D-Navier-Stokes-type system (but with only two components). Surprisingly, this limit system does not depend on the thermal diffusivity $\nu' > 0$. In this talk we explain how to modify the initial data in order to obtain a limit system that really depends on ν'. We will first present the system, then formally obtain a general limit system that we will validate by choosing unconventional initial data. This limit induces a structure that will enable us to separate the solutions of the initial system into two parts, which we will study separately. The convergence will require new Strichartz estimates.

Michele Coti-Zelati (Imperial College, London)

Stability and entropy maximization in the two-dimensional Euler equations

Abstract. We investigate certain questions arising in two-dimensional statistical hydrodynamics, by relying on principles of entropy maximization for the vorticity of a two-dimensional perfect fluid in a disc. In analogy with the entropy functions used in statistical mechanics and thermodynamics, we show that similar concavity properties hold for the 2d Euler equations

when maximizing entropies at fixed energy levels. The proofs rely on rearrangement inequalities, a modification of the classical min-max principle, and the properties of the Euler-Lagrange equations for the corresponding constrained optimization. As a byproduct, we obtain Lyapunov stability for Onsager solutions arising from a system of point-vortices.

Nejla Nouaili (Université Paris Dauphine)
Singularities in the Complex Ginzburg-Landau equation

Abstract. I will present recent results about the study of singularities for the Complex Ginzburg-Landau (CGL) equation. The cubic CGL equation is the most-studied nonlinear equations in the physics community. It was first derived by Newell and Whitehead in 1969 modeling the development of instability in fluid convection problems. The study of singularity formation for CGL equation has received a lot of attention in many works. Typically, we refer to Stewartson and Stuart 1971 for the description of an unstable plane Poiseuille flow. The rigorous proof of the existence of blowup solutions for the CGL equation remained an open question for a long time. I will present constructive examples of finite-time blow-up solutions to the CGL equation. This talk is based on results obtained in collaboration with J.K.Duong and H.Zaag.

Session 55, March 22, 2024 in London (Imperial College London)

Pawel Duch (Adam Mickiewicz University)
PDE construction of fractional Phi4 model of Euclidean quantum field theory in full subcritical regime

Abstract. We present a construction of the Gibbs measure of the fractional Phi4 model of Euclidean quantum field theory in three-dimensions. The measure is obtained as a perturbation of the Gaussian measure with covariance given by the inverse of a fractional Laplacian. Since the Gaussian measure is supported in the space of Schwartz distributions and the quartic interaction potential of the model involves pointwise products, to construct the measure it is necessary to solve the so-called renormalization problem. To this end, we study the stochastic quantization equation, which is a nonlinear parabolic PDE driven by the white noise. We prove a certain a priori estimate for solutions of this equation using the flow equation approach to singular stochastic PDEs and the maximum principle. We consider the

entire range of powers of the fractional Laplacian for which the model is subcritical (i.e. super-renormalizable).
Based on a joint work with M. Gubinelli and P. Rinaldi.

Daniel Han-Kwan (CNRS, Université de Nantes)
From Hartree to Vlasov–Benney

Abstract. I will discuss a derivation of Vlasov–Benney — a Vlasov equation with singular force field — as the semiclassical limit of a (mollified) cubic Hartree equation. The limit is justified for initial data satisfying an appropriate stability condition. This is a joint work with Thomas Chaub and Frederic Rousset (both from Universite Paris-Saclay).

Marjolaine Puel (CYU)
Fractional diffusion approximation for kinetic equations

Abstract. After a short introduction to kinetic equations, I will explain the principle or diffusion approximation which justifies the fact that the solution of a kinetic equation is approximated by a equilibrium profile with a density satisfying a macroscopic equation. I will then focus on the Fokker–Planck equation with heavy tail equilibrium handled by a spectral method.

Tommaso Rosati (University of Warwick)
Lower bounds to Lyapunov exponents of stochastic PDEs

Abstract. Inspired by problems from fluid dynamics, we introduce an approach to obtain lower bounds for Lyapunov exponents of stochastic PDEs. Our proof relies on the introduction of a novel Lyapunov functional for the projective process associated to the equation, based on the study of dynamics of the energy median and on a notion of non-degeneracy of the noise that leads to high-frequency stochastic instability. Joint work with M. Hairer, and in progress with M. Hairer, S. Punshon-Smith and J. Yi.

Session 54, December 15, 2023 in Paris (Institut Henri-Poincaré)

Anne-Sophie De Suzzoni (Ecole polytechnique)
Stability of thermodynamic equilibria for the Hartree-Fock equation with exchange term

Abstract. In the Hartree-Fock equation, which models the evolution of a particle system under symmetry assumptions, the energy exchange term between particles is often neglected in favor of the so-called mean field term. Indeed, under certain structural assumptions about the interaction between particles, these two terms corroborate each other, as is the case for point interaction potentials, i.e., when the Hartree-Fock equation reduces to the Schrödinger equation. On the other hand, the distinction between these two terms has no impact on the locally well-posedness of the equation. Nevertheless, for the global problem, and especially for the problem of asymptotic stability of non-localized equilibria of the equation, the two terms play a very different role and modify the analysis of the linearized equation around the equilibrium under consideration. This talk will present the Hartree-Fock equation with exchange term, its heuristic derivation and its associated equilibria. Finally, a result on the asymptotic stability of thermodynamic equilibria will be presented. This is a collaborative result with Charles Collot (CYU), Elena Danesi (Padova) and Cyril Malézé (Ecole Polytechnique).

Alix Deleporte (Université Paris-Saclay, Orsay)

Semiclassical analysis of free fermions

Abstract. To each orthogonal projector of finite rank N on $L^2(R^d)$ is associated a point process on R^d with N points, which gives the joint probability density of fermions that fill the image of the projector.

The study of the statistical properties of these fermions, in the large N limit, is linked to semiclassical spectral theory problems, some of them well studied (the Weyl law gives a law of large numbers), some of them new. In particular, the behaviour of the variance is linked with the properties of commutators involving spectral projectors, which are not so well understood.

In this talk, I will present my work in collaboration with Gaultier Lambert (KTH) on this topic.

Jeffrey Galkowski (University College London)

The finite element method in high frequency scattering: non-uniform meshes defined by ray-dymanics.

Abstract. One of the most classical ways to numerically approximate the solution to high frequency scattering problems is the finite element method (FEM). In this method, one typically uses piecewise polynomials of some fixed degree p and a mesh-width h to approximate the solution. The fun-

damental questions is then: how should h be chosen (as a function of the frequency, k) so that the error in the numerical solution is small?
It has been known since the seminal work of Babuska and Ihlenberg that the natural conjecture of $hk \ll 1$ is not sufficient. Instead, one must require that $(hk)^{2p}\rho(k) \ll 1$ to maintain constant relative error, where $\rho(k)$ is the norm of the relevant resolvent. In this talk, we will show that this condition can be substantially weakened by using a non-uniform mesh which takes advantage of the fact that errors are concentrated in some regions rather than others.

Jean Lagacé (King's College London)
Explicit class of cut-and-project sets biLipshitz equivalent to lattices

Abstract. How close is a uniformly discrete set to being a lattice? A measure of closeness is biLipschitz (BL) equivalence, and a surprising result by Burago–Kleiner and McMullen at the turn of the millennium is that they are not all, in fact, all BL equivalent to a lattice. Amongst a special class called *cut-and-project* sets, the question is still open: it is known that a large (in the sense of measure) but small (in the sense of Baire categories) and not explicit set of cut-and-project sets are BL equivalent to a lattice, and no counterexample exists. The main method comes from uniform estimates on statistics for point counting. Refining these methods, we make this 'large-but-small' class explicit and show that we cannot get further from simple point counting statistics. The proof relies on a careful analysis of the relation between lattice-point counting and diophantine properties of lattices.
This is joint work with Henna Koivusalo (Bristol).

Session 53, October 20, 2023 in London (King's College London)

Frédéric Bayart (Université Clermont-Auvergne)
Compact and Schatten class composition operators on the Hardy space of Dirichlet series

Abstract. In 1999, Gordon and Hedenmalm determined the composition operators acting on the Hilbert Hardy space of Dirichlet series. This motivated many subsequent works on the properties of these operators. In this talk, we shall present the latest developments regarding compactness and membership to Schatten classes of these operators. In particular we shall describe a geometric sufficient condition.

This talk is partly based on a joint work with Athanasios Kouroupis (NTNU, Trondheim).

Nicolas Lerner (Sorbonne Université)
Signal Theory and Quantum Mechanics

Abstract. The Wigner distribution is a 'quasi-probability' of key importance in Signal Theory. We shall start by showing that this tool is in fact closely connected to Quantum Mechanics, in particular to Hermann Weyl's quantization. Various formulas due to H.Weyl give a larger perspective to the integrals of the Wigner distribution on convex subsets of the phase space and we shall study several examples, starting with discs and ellipses. Furthermore, we shall examine the case of the quarter-plane and will prove that the 1988 Flandrin's conjecture is incorrect: contrarily to that conjecture, the integral of the Wigner distribution on a convex subset of the plane could be larger than 1. We will provide some details on the proofs, which are mostly grounded on the study of special functions linked to the convex subset under scope.

Lucia Scardia (Heriott-Watt University)
Equilibrium measures for nonlocal energies: the effect of anisotropy

Abstract. Nonlocal energies are continuum models for large systems of particles with long-range interactions. Under the assumption that the interaction potential is radially symmetric, several authors have investigated qualitative properties of energy minimisers. But what can be said in the case of anisotropic kernels?
I will present some results and partial answers in this direction obtained in collaboration with Maria Giovanna Mora and Luca Rondi, and with José Antonio Carrillo, Joan Mateu, Joan Verdera and Riccardo Cristoferi.

Sergei Treil (Brown University)
The matrix A_2 conjecture fails, or $3/2 > 1$

Abstract.
The matrix A_2 condition on the matrix weight W

$$[W]_{A_2} := \sup_I \left\| \langle W \rangle_I^{1/2} \langle W^{-1} \rangle_I^{1/2} \right\|^2 < \infty$$

where supremum is taken over all intervals $I \subset \mathbb{R}$, and

$$\langle W \rangle_I := |I|^{-1} \int_I W(s) \mathrm{d}s,$$

is necessary and sufficient for the Hilbert transform T to be bounded in the weighted space $L^2(W)$.

It was well known since early 90s that $\|T\|_{L^2(W)} \gtrsim [W]_{A_2}^{1/2}$ for all weights, and that for some weights $\|T\|_{L^2(W)} \gtrsim [W]_{A_2}$. The famous A_2 conjecture (first stated for scalar weights) claims that the second bound is sharp, i.e.

$$\|T\|_{L^2(W)} \lesssim [W]_{A_2}$$

for all weights.

After some significant developments (and some prizes obtained in the process) the scalar A_2 conjecture was finally proved: first by J. Wittwer for Haar multipliers, then by S. Petermichl for Hilbert Transform and for the Riesz transforms, and finally by T. Hytönen for general Calderón–Zygmund operators.

However, while it was a general consensus that the A_2 conjecture is true in the matrix case as well, the best known estimate, obtained by Nazarov–Petermichl–Treil–Volberg (for all Calderón–Zygmund operators) was only $\lesssim [W]_{A_2}^{3/2}$.

But this upper bound turned out to be sharp! In a recent joint work with K. Domelevo, S. Petermichl and A. Volberg we constructed weights W such that

$$\|T\|_{L^2(W)} \gtrsim [W]_{A_2}^{3/2},$$

so the above exponent $3/2$ is a correct one.

In the talk I'll explain motivations, history of the problem, and outline the main ideas of the construction. The construction is quite complicated, but it is an "almost a theorem" that no simple example is possible.

Joint work with K. Domelevo, S. Petermichl and A. Volberg.

Session 52, June 23, 2023 in Paris (Institut Henri-Poincaré)

Matteo Capoferri (Heriot-Watt University)

Curl and asymmetric pseudodifferential projections

Abstract. In my talk I will present a new approach to the spectral theory of systems of PDEs on closed manifolds, developed in a series of recent papers by Dmitri Vassiliev (UCL) and myself, based on the use of pseudodifferential projections. After discussing the general theory, I will turn to the (non-elliptic) operator curl, and explain how our techniques offer a new pathway to the study of spectral asymmetry.

Frédéric Klopp (Sorbonne Université)

The ground state of a system of interacting fermions in a random field: localization, entanglement entropy, ...

Abstract. Transport in disordered solids is a phenomenon involving many actors. The motion of a single quantum particle in such a solid is described by a random Hamiltonian. Transport involves many interacting particles, usually, a small fraction of the particles present in the material. One striking phenomenon observed and proved in disordered materials is localization: disorder can prevent transport! While this is quite well understood at the level of a single particle, it is much less clear what happens in the case of many interacting particles. Physicist proposed a number of tools (exponential decay of finite particle density matrices, entanglement entropy, etc) to discriminate between transport and localization. Unfortunately, these quantities are very difficult to control mathematically for "real life" models. We'll present a toy model where one can actually get a control on various of these quantities at least for the ground state of the system. The talk is based on the PhD theses of and joint work with N. Veniaminov and V. Ognov.

Jacques Smulevici (Sorbonne Université)

Non-linear periodic waves on the Anti-de-Sitter spacetimes

Abstract. I will present recent results obtained in collaboration with Athanasios Chatzikaleas concerning the construction of periodic in time solutions for various semi-linear nonlinear wave equations on the Anti-de-Sitter spacetimes, as well as an analysis of their stability properties. This question is movatived by the analysis of the Einstein equations in a neighborhood of the Anti-de-Sitter spacetimes, so the talk will start with a general introduction to this problem before a presentation of the results. The proofs are essentially based on black box theorems due to Bambusi-Paleari (for the existence) and Bambusi-Nekhoroshev (for the exponential time stability) together with a detailed Fourier analysis of the different models.

Zoe Wyatt (King's College, London)
Global stability of Kaluza-Klein spacetimes

Abstract. Spacetimes formed from the cartesian product of Minkowski space and a flat torus play an important role as toy models for theories of supergravity and string theory. In this talk I will discuss a result, joint with Cécile Huneau and Annalaura Stingo, showing the nonlinear stability of such a Kaluza-Klein spacetime. I will also explain how our result is connected to some earlier work of Witten.

Session 51, March 30, 2023 in London (University College London)

Blanche Buet (Université Paris Saclay, Orsay),
A varifold perspective on discrete surfaces.

Abstract. Continuous definitions (such as those of surface, regularity, dimension, curvatures ...) generally cannot be readily given a discrete counterpart. Moreover, this discrete counterpart is generally not unique and highly scale-dependent. There are multiple ways of developing a theory for discrete surfaces and the choice of an appropriate framework is directly related to the kind of discrete data we aim to process, and for which purpose i.e. the kind of surfaces we try to model.
We propose to focus on unstructured data in the sense that we do not have any underlying parametrization or topological information associated with our data, a typical example being point cloud data (e.g. obtained from scan acquisition) or different kinds of diffuse approximations (as in MRI for instance). Our motivation is twofold: first, a large range of data-types initially come without any parametrization information. Moreover, let us point out that the construction of such a parametrization (for instance the definition of a triangulation starting from a point cloud) is a challenging active topic in itself, and a better understanding of unstructured data is an essential pre-processing step.
Geometric measure theory offers a particularly well-suited framework for the study of such unstructured discrete surfaces. The long-standing Plateau problem has given birth to several different weakenings of the notion of surface. While their common purpose was to gain compactness while preserving mass/area continuity (or lower semi-continuity at least), they actually provide consistent settings for developing a theory of discrete surfaces, as we

intend to explain in this talk.
Joint work with: Gian Paolo Leonardi (Trento), Simon Masnou (Lyon) and Martin Rumpf (Bonn).

Yann Chaubet (University of Cambridge),
Closed geodesics, intersection numbers and Ruelle resonances.

Abstract. On a closed negatively curved surface, Margulis gave the asymptotic growth of the number of closed geodesics of bounded lengths, when the bound goes to infinity. In this talk, we will investigate such an asymptotic growth for closed geodesics of which certain intersection numbers are prescribed. We will explain the strategy of proof, of which one of the main characters is a dynamical scattering operator. The latter is closely related to the resolvent of the geodesic flow, which will lead us to make use of the theory of Pollicott-Ruelle resonances — the spectral theory of the generator of the geodesic flow.

Jonathan Fraser (University of St Andrews)
Fourier analytic tools in geometric measure theory and dimension interpolation.

Abstract. The Fourier transform of a measure in Euclidean space encodes much geometric and analytic information about the measure - but can be difficult to study. I will discuss some problems in geometric measure theory where Fourier analysis is used effectively and also introduce a new 'dimension interpolation' approach.

Rachid Zarouf (Aix-Marseille Université),
Explicit counterexamples to Schäffer's conjecture, powers of a Blaschke factor and Jacobi polynomials.

Abstract. We prove results that we found on our way to a deeper understanding of Schäffer's conjecture about inverse operators. In 1970 J.J. Schäffer proved that for any invertible $n \times n$ matrix T and for any operator norm $\|\cdot\|$ the inequality

$$|\det T| \left\|T^{-1}\right\| \leq \mathcal{S} \left\|T^{-1}\right\|^{n-1}$$

holds with $\mathcal{S} = \mathcal{S}(n) \leq \sqrt{en}$. He conjectured that in fact this inequality holds with an $\mathcal{S}$ independent of n. This conjecture was refuted in the early

90's by E. Gluskin, M. Meyer and A. Pajor who have shown that for certain $T = T(n)$ the inequality can only hold when $\mathcal{S}$ is growing with n. Subsequent contributions of J. Bourgain and H. Queffélec provided increasing lower estimates on $\mathcal{S}$. Those results rely on probabilistic and number theoretic arguments for the existence of sequences $T(n)$ with growing $\mathcal{S}$. Constructive counterexamples to Schäffer's conjecture were not available since 1995. In this talk we propose a new and entirely constructive approach to Schäffer's conjecture. As a result, we present an explicit sequence of Toeplitz matrices T_λ with singleton spectrum $\{\lambda\} \subset \mathbb{D} \setminus \{0\}$ such that $\mathcal{S} \geq c(\lambda)\sqrt{n}$. A key ingredient in our approach will be to investigate l_p-norms of Fourier coefficients of powers of a Blaschke factor, which is an interesting and well-studied topic in its own right, initiated by J-P. Kahane in 1956. Finally, on our way, we prove new estimates for the asymptotic behavior of Jacobi polynomials with varying parameters and we highlight some flaws in the established literature on this topic.
This is based on a joint work with Oleg Szehr.

Session 50, December 9, 2022 in Paris (Institut Henri-Poincaré)

Isabelle Chalendar (Université Gustave Eiffel),
Weighted composition operators on spaces of holomorphic functions: motivation and spectral properties.

Abstract. Firstly we will recall some links between the description of isometries and weighted composition operators and explain the link between universal operators and composition operators. Then we will investigate the spectral properties of such operators on the Fréchet space $\mathrm{Hol}(\mathbb{D})$ of holomorphic functions on the unit disc $\mathbb{D}$, showing the importance of the Denjoy–Wolff point of the analytic symbol associated with the composition part. We then deduce some spectral properties on various Banach spaces which embeds continuously in $\mathrm{Hol}(\mathbb{D})$.

Mahir Hadzic (University College London),
Selfsimilar collapse for selfgravitating fluids.

Abstract. A basic model of a star is obtained by coupling the compressible Euler equations to Newtonian gravity, which gives the well-known gravitational Euler-Poisson system. In the relativistic context, the relevant model

is the Einstein-Euler system. We give an overview of recent works on the existence of self-similar imploding stars, which correspond to finite time singularities that form from smooth initial data and lead to blow-up of the fluid density. This talk is based on joint works with Y. Guo, J. Jang, and M. Schrecker.

Pierre-François Rodriguez (Imperial College London),
Scaling in low-dimensional long-range percolation models.

Abstract. The talk will present recent progress towards understanding the critical behavior of 3-dimensional percolation models exhibiting long-range correlations. The results rigorously exhibit the scaling behavior of various observables of interest and are consistent with scaling theory below the upper-critical dimension (expectedly equal to 6). This confirms various predictions by physicists based on non-rigorous renormalization group arguments, notably that of Weinrib-Halperin concerning the value of the correlation length exponent and of Fisher's scaling relation for models in this class.

Session 49, October 21, 2022 in London (Queen Mary University of London)

Evgueni Abakoumov (Université Gustave Eiffel),
Chui's conjecture and approximation by simple partial fractions in Bergman spaces.

Abstract. C. K. Chui conjectured in 1971 that the average gravitaional field strength in the unit disk due to unit point masses on its boundary was the smallest when these point masses were equidistributed on the circle. We will present an elementary solution to the analogous minimization problem for weighted L^2 norms, and discuss related questions concerning approximation of holomorphic functions by simple partial fractions. This is joint work with A. Borichev and K. Fedorovskiy.

Jean-Claude Cuenin (Loughborough University),
Effective bounds on scattering resonances.

Abstract. The celebrated Weyl law describes the asymptotic distribution of eigenvalues of the Laplacian on a compact manifold. Scattering resonances are analogues of eigenvalues when the underlying manifold is non-compact. The simplest case concerns Schrödinger operators $-\Delta + V$ on Euclidean space $\mathbb{R}^d$ with compactly supported potential V. The object of interest is the resonance counting function $n_V(r)$, that is, the number of resonances in a disk of radius r. In dimensions greater than one, asymptotics are known only in a few special cases. The topic of this talk are polynomial upper bounds on the resonance counting function. These have a long history, starting in the 80's with work of Melrose. The sharp upper bound $n_V(r) \leq C_V r^d$ was proved by Zworski. In this talk I will present effective versions of this upper bound for non compactly supported potentials. Effective means that C_V does not depend on V itself but only on some weighted norms. The proof of this result features a combination of harmonic, functional and complex analysis.

Louise Gassot (University of Basel),

Zero-dispersion limit for the Benjamin-Ono equation on the torus.

Abstract. We discuss the zero-dispersion limit for the Benjamin-Ono equation on the torus given a single well initial data. We prove that there exist approximate initial data converging to the initial data, such that the corresponding solutions admit a weak limit as the dispersion parameter tends to zero. The weak limit is expressed in terms of the multivalued solution of the inviscid Burgers equation obtained by the method of characteristics. We construct our approximation by using the Birkhoff coordinates of the initial data, introduced by Gérard, Kappeler and Topalov.

Terry Lyons (University of Oxford),

From the mathematics of rough paths to more scalable data science

Abstract. The mathematics of rough path theory creates a framework for understanding the interactions of complex and highly oscillatory systems and generalises the Newtonian framework of controlled differential equations to include rough multimodal evolving systems. A key feature of this theory is the development of a strong analytic theory capturing in concrete terms, the space of (polynomial) functions on these spaces of paths. This work was built on the ideas of KT Chen who studied these spaces of functions to develop a co-homology theory on loop space. The core analysis came from LC Young. The generating function for these polynomial functions on path space is known as the signature, and it was established by Hambly and

Lyons (Annals of Math 2010) that the signature of the path is a complete invariant of a path of finite length modulo the appropriate notion of reparametrisation. Boedihardjo and ... (Advances in Maths 2016) extended this result to rough paths.
These results provide a new perspective and a graded feature set for describing complex streamed data. The first few terms in this series expansion allow high quality local descriptions of streams. These features are expensive to compute. But crucially for machine learning, they only need to be computed once and can be used in every training cycle. In this way they can the form the basis for much more scalable machine learning algorithms. (Morrill, James, Cristopher Salvi, Patrick Kidger, and James Foster. Neural rough differential equations for long time series. In International Conference on Machine Learning, pp. 7829-7838. PMLR, 2021). We will survey this space.

Session 48, June 27, 2022 in Paris (Institut Henri-Poincaré)

David Bate (University of Warwick),
Characterising rectifiable metric spaces using tangent spaces.

Abstract. This talk will present a new characterisation of rectifiable subsets of a complete metric space in terms of local approximation, with respect to the Gromov-Hausdorff distance, by finite dimensional Banach spaces. This is a significant generalisation of a theorem of Marstrand and Mattila of classical geometric measure theory.
After a gentle introduction to analysis on metric spaces, this talk will recall the relevant material from classical GMT. We will then present the main ideas and challenges behind the proof of the new theorem.

Stéphane Jaffard (Université Paris-Est),
Multivariate multifractal analysis : New interplays between fractal geometry, probability and functional analysis.

Abstract. Multifractal analysis quantifies the fluctuations of the pointwise regularity of functions, or measures, through the estimation of their multifractal spectrum, which encapsulates the fractional dimensions of their singularity sets. Wavelet techniques supply robust tools in order to perform multifractal analysis and became a standard signal processing method

for classification or model selection. In view of recent real-world applications that rely on the joint (multivariate) analysis of collections of signals or images, the need for extensions to multivariate settings became a major challenge. We will describe the theoretical foundations of multivariate multifractal analysis, which proposes to quantify the "correlations" between the singularity sets of several functions. We will mention several open problems motivated by this approach.

Sabine Boegli (Durham University),

On the discrete eigenvalues of Schrödinger operators with complex potentials.

Abstract. In this talk I shall present constructions of Schrödinger operators with complex-valued potentials whose spectra exhibit interesting properties. One example shows that for sufficiently large p, the discrete eigenvalues need not be bounded in modulus by the L^p norm of the potential. This is a counterexample to the Laptev-Safronov conjecture (Comm. Math. Phys. 2009). Another construction proves optimality (in some sense) of generalisations of Lieb-Thirring inequalities to the non-selfadjoint case - thus giving us information about the accumulation rate of the discrete eigenvalues to the essential spectrum. This talk is based on joint works with Jean-Claude Cuenin and Frantisek Stampach.

Fabrice Planchon (Sorbonne Université),

Global existence for semilinear wave and Schrödinger equations on domains: what we (don't) know.

Abstract. I will review recent developpements on energy subcritical and critical equations on domains, with emphasis on how the geometry (of the domain and its boundary) may influence both local Cauchy theory (through linear estimates) and global Cauchy theory (through nonlinear a priori estimates and non-concentration arguments). This includes joint works (over an extended period of time) with N. Burq, O. Ivanovici, G. Lebeau, C. Laurent and B. Pausader.

Session 47, December 13, 2019, in Paris (Institut Henri-Poincaré)

Note: important strikes in public transportation led to the cancellation of some of the scheduled talks. Talks were rescheduled for the Spring 2020 session.

Karl-Mikael Perfekt (University of Reading),
Plasmonic eigenvalue problem for corners.

Abstract. We consider the plasmonic eigenvalue problem for 2D domains having a curvilinear corner, studying the spectral theory of the Neumann–Poincaré operator of the boundary. We will see that the corner produces absolutely continuous spectrum of multiplicity 1. The embedded eigenvalues are discrete. In particular, there is no singular continuous spectrum.

Session 46, October 4, 2019, in London (Imperial College London)

Asma Hassannezhad (University of Bristol),
On a relation between Steklov and Laplace spectra on a manifold.

Abstract. On a relation between Steklov and Laplace spectra on a manifold Abstract: The Dirichlet–to–Neumann operator is a first-order elliptic pseudodifferential operator. It acts on smooth functions on the boundary of a Riemannian manifold and maps a function to the normal derivative of its harmonic extension. The eigenvalues of the Dirichlet–to–Neumann map are also called Steklov eigenvalues. It has been known that the geometry of the boundary has a strong influence on the Steklov eigenvalues. In this talk, we show that for any k, the kth Steklov eigenvalue is comparable to the square root of the kth eigenvalue of the Laplacian on the boundary. Our results, in particular, give interesting geometric lower and upper bounds for Steklov eigenvalues.
This is joint work with Bruno Colbois and Alexandre Girouard.

Luc Nguyen (University of Oxford),
Existence and uniqueness of Green's functions to a nonlinear Yamabe problem.

Abstract. On a Riemannian manifold (M, g), the Green's function G_p for the conformal Laplacian with a given pole p in M has the properties that $G_p^{\frac{4}{n-2}} g$ is a complete asymptotically flat metric with zero scalar curvature on $M \setminus \{p\}$. We discuss sharp existence and uniqueness of similar objects when the scalar curvature is replaced by other fully nonlinear conformal curvature quantities.
This is joint work with Yanyan Li.

Nikolai Nikolski (Université de Bordeaux),
Why there exist no Riesz bases or frames positive on a set?

Abstract. It is shown that l^2 weighted signs of any frame (or unconditional basis) in an L^2 space with respect to a continuous measure are "well distributed" on the unit circle. In particular, there is no frame or unconditional basis which are sectorial (in particular, positive) on a set of positive measure. Similar results are obtained for unconditional bases in reflexive Banach lattices, in particular, in L^p spaces, $1 < p < \infty$.
This is joint work with A.Volberg.

Julien Sabin (Université Paris-Sud)),
The Hartree and Vlasov equations at positive density.

Abstract. We consider the Hartree equation around a translation-invariant background, and show that the limit of the Wigner transforms of its solutions as the Planck constant goes to zero converge to solutions to the nonlinear Vlasov equation around the classical version of the translation-invariant background. We also discuss the well-posedness of this Vlasov equation at positive density.
This is joint work with M. Lewin.

Session 45, June 28, 2019, in Paris (Institut Henri-Poincaré)

Sophie Grivaux (CNRS and Université de Lille),
Fourier coefficients of continous measures on the Furstenberg sequence.

Abstract. I will explain how to construct continuous probability measures on the unit circle which have the property that the modulus of their Fourier coefficients on the Furstenberg sequence $\{2^k 3^l \ ; \ k, l \geq 1\}$ is bounded away from zero. This answers in the negative a Conjecture of R. Lyons, motivated by the Furstenberg Conjecture concerning $\times 2$ and $\times 3$ invariant probability measures on the circle. This is joint work with Catalin Badea (Lille).

Andreas Hartmann (Université de Bordeaux),
Random interpolating sequences in Dirichlet spaces.

Abstract. A prominent problem in complex analysis is the description of interpolating sequences in spaces of analytic functions. This problem is related to sampling problems which aim at the reconstructionof a given

function from discrete samples, but also plays a rôle in operator theory or in control theory. Nowadays, interpolating sequences are described in many of the most important spaces of holomorphic functions, like for instance Hardy, Bergman and (Bargmann-)Fock spaces. In the case of the Dirichlet space, which is a subspace of the Hardy space and which is useful e.g. when studying weighted shift operators, the description of interpolating sequences is based on capacities and thus rather hard to check. In such a situation, it is natural to ask whether there are "generic situations", which means that one looks for a framework in which a sequence picked at random is interpolating. In this talk I will discuss random interpolating sequences in weighted Dirichlet spaces $\mathcal{D}_\alpha$, $0 \leq \alpha \leq 1$, and show that in such a random situation, conditions which are known to be significantly weaker in the deterministic setting already imply interpolation. More precisely, it follows from our results that almost sure interpolating sequences for $\mathcal{D}_\alpha$ are exactly the almost sure separated sequences when $0 \leq \alpha < 1/2$ (which includes the Hardy space $H^2 = \mathcal{D}_0$), and they are exactly the almost sure zero sequences for $\mathcal{D}_\alpha$ when $1/2 < \alpha < 1$. I will also discuss the situation in the classical Dirichlet space $\mathcal{D} = \mathcal{D}_1$ where we get an almost optimal result in a sense.

Lauri Oksanen (University College London),
Inverse problems for hyperbolic PDE.

Abstract. We discuss some recent results concerning coefficient determination problems for hyperbolic partial differential equations, both linear and non-linear. In the linear case the known results are essentially confined to ultrastatic spacetimes, whereas some non-linear cases allow for much more general Lorentzian geometries.

Session 44, March 22, 2019, in London (King's College, London)

Manuel del Pino (University of Bath),
Gluing methods for Vortex dynamics in Euler flows.

Abstract. We consider the two-dimensional Euler flow for an incompressible fluid confined to a smooth domain. We construct smooth solutions with concentrated vorticities around points which evolve according to the Hamiltonian system for the Kirkhoff-Routh energy. The profile around each point resembles a scaled finite mass solution of Liouville's equation. We discuss extensions of this analysis to the case of vortex filaments in 3-dimensional space.

Stefanie Petermichl (Université de Toulouse),
Sparse domination and dimensionless estimates for the Riesz vector.

Abstract. It is known since the 1970s, formulated in the work by Gundy and Varopoulos, that Riesz transforms have stochastic representations using the background noise process and harmonic extensions. On the other hand, their point-wise domination by so-called sparse operators is known since 2015 by Nazarov-Lerner, Lacey, independently. These recent sparse domination principles are based on stopping cubes and carry dimensional constants. Through a probabilistic argument, a trajectory-wise sparse domination with continuous parameter can be obtained, thus avoiding all occurrences of dimensional constants. We give a new proof of a dimensionless L^p estimate for the Bakry Riesz vector on Riemannian manifolds with bounded geometry. Our proof gives a significantly stronger conclusion in that it gives new dimensionless L^p estimates, even under a reasonable change of measure.

Maria Reguera (University of Birmingham),
Sparse forms for Bochner-Riesz operators.

Abstract. Sparse operators are positive dyadic operators that have very nice boundedness properties. The L^p bounds and weighted L^p bounds with sharp constant are easy to obtain for these operators. In the recent years, it has been proven that singular integrals (cancellative operators) can be pointwise controlled by sparse operators. This has made the sharp weighted theory of singular integrals quite straightforward. The current efforts focus in understanding the use of sparse operators to bound rougher operators, such a oscillatory integrals. Following this direction, our goal in this talk is to describe the control of Bochner-Riesz operators by sparse operators.

Nikolay Tzvetkov (Université Cergy-Pontoise),
Solving the 4NLS with white noise initial data.

Abstract. We construct global-in-time singular dynamics for the (renormalized) cubic fourth order nonlinear Schroedinger equation on the circle, having the white noise measure as an invariant measure. We achieve this goal by using a random gauge transform which leads to random Bourgain spaces. The singular component of the solution consists of arbitrarily high powers of the random initial data. Consequently the result is somehow similar in spirit to the modified scattering results occurring in the study of small localised solutions. This is a joint work with Tadahiro Oh and Yuzhao Wang.

Session 43, December 14, 2018, in Paris (Institut Henri-Poincaré)

Frédéric Bernicot (Université de Nantes),
A bilinear principle of orthogonality.

Abstract. The Rubio de Francia inequality allows us to describe the orthogonality in L^p (for $p > 2$) for Fourier projections associated to disjoint intervals (in frequency). We aim to describe a bilinear analogue of this principle, by considering bilinear Fourier projections and prove the bilinear counterpart of these inequalities. In the linear theory, the L^2 case is trivial as a direct consequence of Plancherel theorem. In the bilinear setting, there is no such easy first inequalities and any kind of inequalities is difficult to prove. It requires new arguments with respect to the linear case. This is joint work with Cristina Benea and Marco Vitturi.

Anne de Bouard (École polytechnique),
Stochastic effects in liquid crystal flows.

Abstract. We investigate existence and uniqueness of solutions for the liquid crystal flow driven by colored noise, on the two-dimensional torus (the so called Eriksen-Leslie equation). The PDE system couples a Navier Stokes equation for the fluid velocity of the liquid crystal and a gradient flow for the director, a vector field with values on the three dimensional sphere. The stochastic perturbations are white in time, and arise only on the director equation. The solutions we are looking for are in the spirit of the solutions found by Struwe for the deterministic harmonic map flow : they are piecewise regular, but may present singularities at discrete times and discrete points in space, in which the gradient of the director and the fluid velocity may concentrate in the L^2 space. This is a joint work with Antoine Hocquet and Andreas Prohl.

Ilya Goldsheid (Queen Mary University of London),
Invariant measure equation for random walks on random environments on a strip.

Abstract. Environment viewed from the particle is a powerful method of analyzing random walks (RW) in random environment (RE). The well known fact is that in this setting the environment process is a Markov chain on the set of environments. In the talk I shall discuss the fundamental question of the existence of the density of the invariant measure of this Markov chain with respect to the measure on the set of environments for RWs on a strip. I shall describe all positive sub-exponentially growing solutions of the corresponding invariant density equation in a deterministic setting and then derive the necessary and sufficient conditions for the existence of the density when the environment is ergodic in both the transient and the recurrent regimes. Time permitting, I'll also discuss the applications of our analysis to the question of positive and null recurrence and to random walks on orbits of a dynamical system. This is joint work with D. Dolgopyat

Alexander Sobolev (University College London),
Formulas of Szegő type for the periodic Schrödinger operator.

Abstract. We prove asymptotic formulas of Szegő type for the periodic Schrödinger operator $H = -\frac{d^2}{dx^2} + V$ in dimension one. Admitting fairly general functions h with $h(0) = 0$, we study the trace of the operator $h\big(\chi_{(-\alpha,\alpha)} E_{(-\infty,\mu)}(H) \chi_{(-\alpha,\alpha)}\big)$, as $\alpha \to \infty$, where $\chi_{(-\alpha,\alpha)}$ is the indicator of the interval $(-\alpha, \alpha)$ and $E_{(-\infty,\mu)}(H)$ is the spectral projection of H for the interval $(-\infty, \mu)$. This is joint work with Bernhard Pfirsch.

Session 42, October 5, 2018, in London (University College London)

Emmanuel Fricain (Université de Lille),
Multipliers between sub-Hardy Hilbert spaces.

Abstract. In this talk, I will discuss multipliers from one sub-Hardy Hilbert space to another. We first examinate the case of model spaces and see that this problem is connected to Carleson measures and Beurling-Malliavin theorem. In a second part of the talk, we will discuss the case of the range of co-analytic Toeplitz operators which are connected to de Branges-Rovnyak spaces. This talk is based on collaborations with A. Hartmann, W. Ross and R. Rupam.

Sandrine Grellier (Université d'Orléans),

Generic colourful tori and inverse spectral transform for Hankel operators.

Abstract. This talk is devoted to explore the regularity properties of an inverse spectral transform for Hilbert–Schmidt Hankel operators on the unit disc. This spectral transform plays the role of action-angles variables for an integrable infinite dimensional Hamiltonian system – the cubic Szegő equation. We investigate the regularity of functions on the tori supporting the dynamics of this system. We prove that generic smooth functions and a G_δ dense set of irregular functions do coexist on the same torus. This is from joint works with Patrick Gérard, Laboratoire mathématiques Orsay.

Tom Korner (University of Cambridge),
Can we characterise sets of strong uniqueness?

Abstract. A closed subset of the circle is said to be of strong uniqueness if it does not support a non-zero measure whose Fourier transform tends to zero at infinity. Debs and Saint Raymond showed that the answer is no (at least for the type of characterisation which have been attempted) and this talk says no even more strongly.

Tom Sanders (University of Oxford),
The Erdös-Moser sum-free set problem.

Abstract. We consider the problem of finding a large subset S of a given set A of integers such that none of the sums of distinct elements of S are in A. A greedy argument gives a logarithmic bound — $|S| \geq c \log |A|$ — and about 10 years ago Sudakov, Szemeredi and Vu made an important breakthrough when they showed one could take $|S|$ to be super-logarithmic. In this talk we shall give an argument to show one can take $|S| = \log^{1+c} |A|$ and sketch some limitations of the method.

Session 41, June 15, 2018, in Paris (Jussieu campus of Sorbonne Université)

Luc Hillairet (Université d'Orléans),
Multiple geometric diffractions at conical points.

Abstract. We study the wave propagation on flat surfaces with conical singularities. After hitting a conical point, the singularities of the wave decompose into two parts : the direct and the diffracted front. These two fronts intersect along two diffractive rays that are limits of non-diffractive ones and

are called geometric diffractive rays. With Andrew Hassell and Austin Ford, we propose a new construction of the propagator near these rays. This new description is well adapted to then studying multiple geometric diffractions (when a geometric diffractive ray hits again a conical point) and eventually the contribution to the wave-trace of any kind of periodic diffractive orbit.

Ari Laptev (Imperial College London),
Spectral properties of some functional-difference operators for mirror curves.

Abstract.

Galina Perelman (Université Paris 12),
Blow up dynamics for the hyperbolic vanishing mean curvature flow of surfaces asymptotic to Simons cone.

Abstract. We consider the hyperbolic vanishing mean curvature flow of surfaces in $\mathbb{R}^8$ asymptotic at infinity to Simons cone:

$$C_4 = \left\{ X = (x_1, \cdots, x_8) \in \mathbb{R}^8 \, , x_1^2 + \cdots + x_4^2 = x_5^2 + \cdots + x_8^2 \right\}.$$

We show that the flow admits finite time blow up solutions $(\Gamma(t))_{0<t\leq T}$ that blow up by concentration of the stationary profile: there exists a smooth minimal surface M asymptotic at infinity to Simons cone such that

$$\Gamma(t) \sim t^{\nu+1} \, M \, , \text{ as } t \to 0 \, ,$$

where ν is an arbitrary large positive number.
This is a joint work with Hajer Bahouri and Alaa Marachli.

Alexander Strohmaier (University of Leeds),
Local and global index for Dirac operators on Lorentzian spacetimes.

Abstract. I will review some recent results on index theory for the Lorentzian Dirac operator on a globally hyperbolic spacetime. I will show how such index theorems can be derived from Lorentzian versions of the local index theorem and I will explain some essential differences to the Riemannian setting. In particular local elliptic theory is replaced by global propagation of singularity estimates. (joint work with C. Baer)

Session 40, March 23, 2018, in London (Queen Mary University of London)

Laurent Baratchart (Inria, Sophia-Antipolis,
Topics in Rational and Meromorphic Approximation.

Abstract. We consider rational and meromorphic approximation with n poles to analytic functions on a compact subset of the domain of analyticity. We shall recast this issue as optimal discretization of logarithmic potentials, and stress some applications to identification and elliptic inverse problems. We will discuss the asymptotic behaviour of poles, when n gets large, for best approximants to certain classes of functions, in connection with interpolation theory, the Adamjan-Arov-Krein theory, and extremal geometric problems from potential theory. We will also speak on error rates along with constructive aspects, and raise some questions in higher dimension.

Leonid Bogachev (University of Leeds),
Liouville-type theorems for the archetypal equation with rescaling.

Abstract. In this talk, we consider a linear functional-integral equation

$$y(x) = \iint_{\mathbb{R}^2} y(a(x-b))\,\mu(\mathrm{d}a, \mathrm{d}b), \qquad x \in \mathbb{R},$$

where μ is a probability measure on $\mathbb{R}^2$; equivalently, $y(x) = \mathbb{E}\{y(\alpha(x-\beta))\}$, with random (α, β) and $\mathbb{E}$ denoting expectation. This is a rich source of various functional and functional-differential equations with rescaling (hence the name *archetypal*), exemplified by the integrated Cauchy equation $y(x) = \mathbb{E}\{y(x-\beta)\}$ and the functional-differential ('pantograph') equation $y'(x) + y(x) = \mathbb{E}\{y(\alpha(x-\gamma))\}$.
Interpreting solutions $y(x)$ as harmonic functions of the associated Markov chain (X_n), we discuss Liouville-type theorems asserting that any bounded continuous solution is constant. For instance, in the case $\alpha \equiv 1$ this is the celebrated Choquet–Deny theorem. In general, results crucially depend on the criticality parameter $K := \mathbb{E}\{\ln|\alpha|\}$; e.g., if $K < 0$ then a Liouville theorem is always true, but the case $K \geq 0$ is more interesting (and difficult). The proofs utilize the iterated equation $y(x) = \mathbb{E}\{y(X_\tau)|X_0 = x\}$ (with a suitable stopping time τ) due to Doob's optional stopping theorem applied to the martingale $y(X_n)$.
This is joint work with Gregory Derfel (Beer Sheva) and Stanislav Molchanov (UNC-Charlotte).

Victor Chulaevsky (Université de Reims),
Smoothness of density of states and localization under long-range interactions with arbitrary disorder: New challenges.

Abstract. Numerous works on spectral and dynamical properties of disordered media, in mathematical and theoretical physics, are based on the assumption of local regularity of the disorder. It was shown in the seminal paper by F. Wegner (1981) that if a finite matrix has independent random diagonal elements with a Lipschitz continuous distribution and its off-diagonal part is non-random, then its eigenvalue distribution is also Lipschitz continuous. The Wegner estimate has been generalized in various directions and applied to many discrete and continuous random Hamiltonians; it is a crucial component of the rigorous proofs of Anderson localization. However, the regularity of the original, local disorder is questionable from the physical perspective. An extension to the models with very singular disorder (e.g., Bernoulli-distributed amplitudes in the so-called continuous alloy models) is much more recent, and the available techniques, surprisingly, fail to apply to the discrete (e.g., lattice) systems.
In contrast to a majority of mathematical works, we consider in the talk alloy models with physically more realistic (infinite-range) site potentials and show that classical results on infinite convolutions of singular probability measures, a subject that has been actively studied in harmonic analysis, probability theory/statistics, and dynamical systems since early 20th century, shed a light on this hard problem and allow one to prove in many cases an infinite smoothness of the finite-volume eigenvalue distribution measure. At the same time, the first results in this direction also raise new interesting and challenging questions.

Stephen Power (University of Lancaster),
Crystal flexibility: methods from Analysis and Commutative Algebra.

Abstract. The stability of crystal lattices was considered by Max Born and coworkers in the 1940s by essentially linear (small vibration) methods for which "it is not necessary to consider the complete elastic spectrum with its innumerable proper frequencies". The analysis of such mechanical modes (a.k.a. zero modes, rigid unit modes) is an ongoing research topic in both mathematics (infinitesimal and combinatorial rigidity) and condensed-matter science (surface modes and topological modes). I shall survey some mathematical rigidity theory and I hope to indicate some open problems and new methods from analysis and commutative algebra. In particular us-

ing algebraic spectral synthesis methods necessary and sufficient conditions have been obtained for the first-order rigidity of a crystallographic bar-joint framework (Kastis and Power, 2018).

Session 39, December 15, 2017 in Paris (Institut Henri-Poincaré)

Sylvie Benzoni-Gavage (Université Claude-Bernard, Lyon 1),
Stability of Hamiltonian periodic waves.

Abstract. The stability of nonlinear periodic travelling waves has been studied intensively in the last fifteen years. Nevertheless, stability criteria can hardly ever be checked analytically for general partial differential equations. The talk will be concerned with two asymptotic regimes in a rather general Hamiltonian framework. More specifically, I will consider waves of either small amplitude or large wavelength. The main purpose will be to show how the expansion of stability criteria reveals non degeneracy conditions for small amplitude waves, and the relationship with the stability of solitary waves for large wavelength waves.

Mark Pollicott (University of Warwick),
Transfer operators, determinants and some applications.

Abstract. The transfer operators we are interested in are linear operators closely related to composition operators on the Hardy Hilbert spaces. They are examples of trace class operators, and one can associate a determinant function, i.e., an entire function of a complex variable. This viewpoint is useful, for example, in studying: (i) Zeros of the Selberg zeta function in geometry; (ii) Numerical estimation of the Hausdorff Dimension of some sets. No prior knowledge will be assumed.

Peter Topping (University of Warwick),
Ricci flow and Ricci limit spaces.

Abstract. Ricci flow theory has been developing rapidly over the last couple of years, with the ability to handle Ricci flows with unbounded curvature finally becoming a reality. This is vastly expanding the range of potential applications. I will describe some recent work in this direction with Miles Simon that shows the right way to pose the Ricci flow PDE in this setting in

order to make applications to the understanding of Ricci limit spaces. (No knowledge of Ricci flow and Ricci limit spaces etc. will be assumed.)

Franck Sueur (Université de Bordeaux),
Controllability of the Navier-Stokes equation in a rectangle with a little help of an interior phantom force.

Abstract. We consider the 2D incompressible Navier-Stokes equation in a rectangle with the usual no-slip boundary condition prescribed on the upper and lower boundaries. We prove that for any positive time, for any finite energy initial data, there exist controls on the left and right boundaries and a distributed force such that the corresponding solution is at rest at the given final time. The distributed force can be chosen arbitrarily small in any Sobolev norm in space, and supported away from the uncontrolled boundaries. This is joint work with Jean-Michel Coron, Frédéric Marbach and Ping Zhang.

Session 38, October 6, 2017 in London (Imperial College London)

Alexander Borichev (Aix-Marseille Université),
Geometry of families of reproducing kernels in Fock spaces.

Abstract. We study families of reproducing kernels in (weighted) Fock spaces. Main questions we are interested in are when such a family is a Riesz basis, a strong Markuschevich basis, and when the biorthogonal family is complete.

Claudia Garetto (Loughborough University),
A survey on hyperbolic equations and systems with multiplicities.

Abstract. In this talk I present some recent results for hyperbolic equations and systems with multiplicities obtained in collaboration with M. Ruzhansky (Imperial College London) and C. Jäh (Loughborough University). Particular attention will be given to the regularity of the coefficients and the role played by the lower order terms.

Gregory Seregin (University of Oxford),
Liouville type theorems for Navier-Stokes equations.

Abstract.The talk is addressed the regularity theory for the Navier-Stokes equations via rescaling and Liouville type theorems. As a related question, Liouville type theorems for steady-state Navier-Stokes equations will be discussed as well.

Armen Shirikyan (Université Cergy-Pontoise),
An elementary introduction to fluctuation relation and fluctuation theorem in chaotic dynamical systems.

Abstract. We begin with a general description of fluctuation relation for stochastic systems. After introducing some simple objects related to the entropy production, we show that the fluctuation relation as proposed by Evans-Searles, Gallavotti-Cohen, and Lebowitz-Spohn is a consequence of the large deviations principle (LDP). We next turn to a class of chaotic dynamical systems and study the validity of LDP and fluctuation relation. Under rather general hypotheses allowing for phase transitions, we prove that the empirical measures satisfy an LDP with a convex good rate function for which a fluctuation relation holds. This is a joint work with N. Cuneo, V. Jaksic, and C.-A. Pillet.

Session 37, June 23, 2017 in Paris (Institut Henri-Poincaré)

Olivier Glass (Université Paris-Dauphine),
Control of the motion of a fluid at low Reynolds number.

Abstract. I will describe a result obtained with Thierry Horsin (Conservatoire national des arts et métiers, Paris), concerning the possibility of prescribing the motion of a zone of a fluid inside a domain, by means of a boundary control. When the fluid is very viscous (and modeled by the stationary Stokes equation), we obtain results proving that prescribing this motion is indeed possible in an approximate way. This relies on an adaptation for the Stokes equation of Runge's theorem concerning the approximation of holomorphic functions by rational functions.

Oana Pocovnicu (Heriot-Watt University),
A two-soliton with transient turbulent regime for a focusing cubic nonlinear half-wave equation on the real line .

Abstract. In this talk we consider a nonlocal focusing cubic half-wave equa-

tion on the real line. Evolution problems with nonlocal dispersion naturally arise in physical settings which include models for wave turbulence, continuum limits of lattice systems, and gravitational collapse. The goal of the talk is to present the construction of an asymptotic global-in-time modulated two-soliton solution of small mass, which exhibits the following two regimes: (i) a turbulent regime characterized by an explicit growth of high Sobolev norms on a finite time interval, followed by (ii) a stabilized regime in which the high Sobolev norms remain stationary large forever in time. This talk is based on joint work with P. Gérard (Orsay, France), E. Lenzmann (Basel, Switzerland), and P. Raphael (Nice, France).

Michael Ruzhansky (Imperial College London),
Very weak solutions to wave equations.

Abstract. In this talk we will discuss the wave type equations with time-dependent very singular (distributional) coefficients. Examples will include the wave equation for the Landau Hamiltonian as well as equations arising in acoustics and in shallow water problems. We present two type or results: for equations with Hölder coefficients (in the spirit of Colombini, de Giorgi, and Spagnolo), and for equations with distributional coefficients (very weak solutions). There appear some interesting phenomena that we will discuss (also numerically). If time permits, we will also give results on for the corresponding wave equations for the sub-Laplacian on stratified Lie groups (e.g. on the Heisenberg group) as well as for higher order operators (such as Rockland operators on graded Lie groups). The talk will be mostly based on different joint works with Claudia Garetto and Niyaz Tokmagambetov.

Joseph Viola (Université de Nantes),
The Hamilton flow and Schrödinger evolution for degree-2 complex-valued Hamiltonians.

Abstract. Given a (complex-valued, degree 2) polynomial on phase space, we can study both the (classical) Hamilton flow and the (quantum) Schrödinger evolution. We discuss the relationship between these two objects: most concretely, we will show how to use the Hamilton flow to find the L^2 operator norm of the Schrödinger evolution, when this evolution operator is compact.

Session 36, March 31, 2017 in London (King's College, London)

Michael Farber (Queen Mary University of London),
Topology of large random spaces.

Abstract. I will discuss probabilistic models generating random simplicial complexes. One is able to predict their topological properties with probability tending to one when the spaces are large, i.e. depend on a growing number of independent random variables.

Evelyne Miot (Université Joseph-Fourier, Grenoble),
Uniqueness and stability for the Vlasov-Poisson system with spatial density in Orlicz spaces.

Abstract. We study uniqueness and stability issues for the Vlasov-Poisson system with spatial density belonging to a certain class of Orlicz spaces. In particular, we provide a quantitative stability estimate for the Wasserstein distance between two weak solutions with spatial density in such Orlicz spaces, in the spirit of Dobrushin's proof of stability for mean-field PDEs. Our proofs are built on the second-order structure of the underlying characteristic system associated to the equation. This is joint work with T. Holding.

Gilles Pisier (Université Pierre-et-Marie-Curie, Paris and Texas A&M University),
Lacunary series in duals of compact groups and generalizations.

Abstract. We will recall some of the classical theory of Sidon sets of characters on compact groups (Abelian or not). We will then give several recent extensions to Sidon sets, randomly Sidon sets and subgaussian sequences in bounded orthonormal systems, following recent work by Bourgain and Lewko, and by the author, both currently available on arxiv. The case of matricial systems, analogous to Fourier-Peter-Weyl series on compact groups, connects the subject to random matrix theory. An unpublished result of Rider (circa 1975) will also be highlighted.

Igor Wigman (King's College London),
Nodal intersections of random toral eigenfunctions against a test curve.

Abstract. This talk is based on joint works with Zeev Rudnick, and Maurizia Rossi.

We investigate the number of nodal intersections of random Gaussian Laplace eigenfunctions on the standard 2-dimensional flat torus ("arithmetic random waves") with a fixed reference curve. The expected intersection number is universally proportional to the length of the reference curve, times the wavenumber, independent of the geometry.
Our first result prescribes the asymptotic behaviour of the nodal intersections variance for generic smooth curves in the high energy limit; remarkably, it is dependent on both the angular distribution of lattice points lying on the circle with radius corresponding to the given wavenumber, and the geometry of the given curve. For these curves we can prove the Central Limit Theorem. We then construct some examples of exceptional "static" curves where the variance is of smaller order of magnitude, and the limit distribution is non-Gaussian.

Session 35, December 9, 2016 in Paris (Institut Henri-Poincaré)

David Chiron (Université de Nice - Sophia Antipolis),
Multiple branches of travelling waves in the Gross-Pitaevskii equation.

Abstract. We consider the Nonlinear Schrödinger or Gross-Pitaevskii equation in the plane. This equation possesses travelling waves solutions, first studied by Jones and Roberts, for speeds between 0 and the speed of acoustic waves $\sqrt{2}$. These travelling waves are well described for c close to 0, where the solution exhibit vortices, and for c close to $\sqrt{2}$, where they are asymptotically described by the Kadomtsev-Petviashvili-I (KP-I) equation. In this talk, we shall give some numerical results showing the existence of other branches of travelling waves, corresponding to excited states, coming from both of the limits c close to $\sqrt{2}$ and c going to 0. This is a joint work with C. Scheid (Nice).

Jimmy Lamboley (Université Paris-Dauphine),
Minimization of the compliance of a connected 1-dimensional set

Abstract. In this talk, we describe the features of the following optimization problem, whose unknown is a connected 1-dimensional set in $\mathbb{R}^2$:

$$\min\{\mathcal{C}(\Sigma) + \lambda \mathcal{H}^1(\Sigma), \Sigma \text{ closed connected subset of } \overline{\Omega}\},$$

where Ω is a fixed open set of $\mathbb{R}^2$, $\lambda > 0$, $\mathcal{H}^1(\Sigma)$ denotes the length of Σ (its 1-dimensional Hausdorff measure), and $\mathcal{C}(\Sigma)$ denotes the compliance of

$\Omega \setminus \Sigma$, that is the opposite of the Dirichlet energy of $\Omega \setminus \Sigma$ (for an external force term $f \in L^2(\Omega)$).
This problem can be interpreted as to find the best location for attaching (on Σ) a membrane Ω subject to a given external force f so as to minimize its compliance. It can be seen as an elliptic PDE version of the average distance problem/irrigation problem which has been extensively studied in the literature, and is also deeply related to the famous Mumford-Shah problem. We particularly focus on the regularity and the topology of minimizers, we prove that they are made of a finite number of smooth curves meeting only by three at 120 degree angles, containing no loop, and possibly touching $\partial\Omega$ only tangentially. We will describe the classical tools and the new ones we developed for this purpose.
This is a joint work with A. Chambolle, A. Lemenant and E. Stepanov.

Alexander Pushnitski (King's College London),
Inverse spectral problem for positive Hankel operators

Abstract. Hankel operators are infinite matrices with entries a_{n+m} depending on the sum of indices. I will discuss an inverse spectral problem for a certain class of positive Hankel operators. The problem appeared in the recent work by P.Gerard and S.Grellier as a step towards description of evolution in a model integrable non-dispersive equation. Several features of this inverse problem make it strikingly (and somewhat mysteriously) similar to an inverse problem for Sturm-Liouville operators. I will describe the available results for Hankel operators, emphasizing this similarity. This is joint work with Patrick Gerard (Orsay).

Gwyneth Stallard (The Open University),
The structure of the escaping set in complex dynamics

Abstract. Complex dynamics concerns the behaviour of points in the complex plane under iteration by a holomorphic function. This talk is particularly concerned with the iterative behaviour of transcendental entire functions such as exponential functions. The escaping set is the set of points that escape to infinity under iteration and plays a key role in complex dynamics. Much research in recent years has been motivated by Eremenko's conjecture that all the components of the escaping set are unbounded and has led to a much deeper understanding of the possible structures of the escaping set. This talk gives an overview of work in this area, particularly of joint work

with Phil Rippon.

Session 34, October 14, 2016 in London (University College London)

Aline Bonami (Université d'Orléans),
Fourier multipliers of the Sobolev space $W^{1,1}$.

Abstract. We will consider Fourier multipliers of the space $W^{1,1}(\mathbb{R}^d)$ (resp. the homogeneous space $\dot{W}^{1,1}(\mathbb{R}^d)$). It is well known that Fourier multipliers of the space $L^1(\mathbb{R}^d)$ coincide with Fourier transforms of bounded measures. The same characterization is valid for Fourier multipliers of $\dot{W}^{1,1}(\mathbb{R}^d)$ when $d = 1$, but the situation is different for $d > 1$. Namely, Poornima proved the existence of other multipliers by using a delicate construction of Ornstein. On the other hand, no non constant homogeneous function of degree 0 is a Fourier multiplier of $\dot{W}^{1,1}(\mathbb{R}^d)$. It was proved recently by Kazaniecki and Wojciechowski that such Fourier multipliers are continous functions.
We will give the analogue in this context of De Leeuw Theorems for Fourier multipliers of $L^p(\mathbb{R}^d)$, that is, prove that the restriction of a Fourier multiplier to some subgroups of $\mathbb{R}^d$ is still a Fourier multiplier. We will also consider extension theorems and give new examples of Fourier multipliers of $W^{1,1}(\mathbb{R}^d)$ and $\dot{W}^{1,1}(\mathbb{R}^d)$.

This is work in progress with Madan and Mohanty (IIT Kanpur, India).

Gabriel Paternain (University of Cambridge),
Recovering a connection from parallel transport along geodesics

Abstract. I will discuss the inverse problem of recovering a unitary connection from the parallel transport along geodesics of a compact Riemannian manifold with strictly convex boundary. It is possible to solve this geometric inverse problem in two disjoint settings: manifolds of negative curvature and manifolds of non-negative curvature. The solutions are based on a range of techniques, including energy estimates, regularity results for the transport equation associated with the geodesic flow and microlocal analysis.

Nadia Sidorova (University College London),
Delocalising the parabolic Anderson model

Abstract. The parabolic Anderson problem is the Cauchy problem for the heat equation on the integer lattice with random potential. It is well-known

that, unlike the standard heat equation, the solution of the parabolic Anderson model exhibits strong localisation. In particular, for a wide class of iid potentials (including Pareto potentials) it is localised at just one point. In the talk, we discuss a natural modification of the parabolic Anderson model on Z, where the one-point localisation breaks down for heavy-tailed Pareto potentials and remains unchanged for light-tailed Pareto potentials, exhibiting a phase transition at the Pareto parameter 2. This is a joint work with Stephen Muirhead and Richard Pymar.

Laurent Stolovitch (Université de Nice-Sophia Antipolis),
Real submanifolds of maximum complex tangent space at a CR singular point
1 Abstract. In this joint work with Xianghong Gong (Madison), we study a germ of real analytic n-dimensional submanifold of $\mathbf{C}^n$ that has a complex tangent space of maximal dimension at a CR singularity. Under some assumptions, it is a perturbation of a quadric and we show its equivalence to a normal form under a local biholomorphism at the singularity. We also show that if a real submanifold is formally equivalent to a quadric, it is actually holomorphically equivalent to it, if a "small divisors condition" is satisfied.

Session 33, June 17, 2016, in Paris (Institut Henri-Poincaré)

Valeria Banica (Université d'Evry),
Collision of almost parallel vortex filaments.

Abstract. We investigate the occurrence of collisions in the evolution of vortex filaments through a system introduced by Klein, Majda and Damodaran and by Zakharov. We first establish rigorously the existence of a pair of almost parallel vortex filaments, with opposite circulation, colliding at some point in finite time. The collision mechanism is based on the one of the self-similar solutions of the model, described in our previous work. We also extend this construction to the case of an arbitrary number of filaments, with polygonal symmetry, that are perturbations of a configuration of parallel vortex filaments forming a polygon, with or without its center, rotating with constant angular velocity. This is a joint work with Erwan Faou and Evelyne Miot.

Julio Delgado (Imperial College London),

Schatten-von Neumann properties on compact manifolds

Abstract. In this talk we present some recent results on the study of Schatten-von Neumann properties for operators on compact manifolds. We will explain the point of view of kernels and full symbols. The special case of compact Lie groups is treated separately. We will also discuss about operators on L^p spaces by using the notion of nuclear operator in the sense of Grothendieck and deduce Grothendieck-Lidskii trace formulas in terms of the matrix-symbol . (This a joint work with Michael Ruzhansky.)

Nader Masmoudi (New York University, USA),
Stability of the 3D Couette Flow

Abstract. We discuss the dynamics of small perturbations of the plane, periodic Couette flow in the 3D incompressible Navier-Stokes equations at high Reynolds number. For sufficiently regular initial data, we determine the stability threshold for small perturbations and characterize the long time dynamics of solutions near this threshold. For rougher data, we obtain an estimate of the stability threshold which agrees closely with numerical experiments. The primary linear stability mechanism is an anisotropic enhanced dissipation resulting from the mixing caused by the large mean shear; the main linear instability is a non-normal instability known as the lift-up effect. Understanding the variety of nonlinear resonances and devising the correct norms to estimate them form the core of the analysis we undertake. Joint work with Pierre Germain and Jacob Bedrossian.

Clément Mouhot (University of Cambridge),
Hölder continuity of solutions to Vlasov-Fokker-Planck type equations with rough coefficients

Abstract. The celebrated De Giorgi-Nash theory about Hölder continuity of solutions to elliptic or parabolic equations with rough –i.e. merely measurable– coefficients in the late 1950s is a cornerstone of modern PDE analysis. We extend this theory to a class of kinetic equation of Vlasov-Fokker-Planck type ("hypoelliptic of type II" in the terminology of Hörmander) where a first-order hyperbolic operator interacts with a partially elliptic operator with rough coefficients. We also extend the theory of Moser about Harnack inequalities for these equations. This is a joint work with F. Golse, C. Imbert and A. Vasseur.

Session 32, March 18, 2016, in London (Queen Mary University of London)

Thierry Levy (Université Pierre et Marie Curie),
The Douglas-Kazakov phase transition.

Abstract. Douglas and Kazakov predicted about twenty years ago that the pure Euclidean Yang-Mills theory on the two-dimensional sphere with structure group $U(N)$ exhibits a phase transition, in the limit where N tends to infinity, when the area of the sphere crosses the critical value π^2. In probabilistic language, this can be expressed as a phase transition for

the Brownian bridge on the unitary group $U(N)$ in the large N limit, when the length of the bridge crosses the same critical value π^2. I will describe this phase transition from two points of view, on one hand by discussing the distribution of the eigenvalues of certain random unitary matrices, and on the other hand by looking for the dominant Fourier modes of the heat kernel on the unitary group. This is joint work with Mylène Maïda (Lille).

Ivan Todorov (Queen's University Belfast),

Operator systems, non-signalling correlations and quantum graph parameters

Abstract. Operator systems have played a central role in Operator Algebra Theory since their introduction in the 1970's. Capturing the features of non-commutative order, they and their morphisms "completely positive maps" have been prominent in C^*-algebra Theory, Operator Space Theory and, lately, Quantum Information Theory. The talk will be centred around their use in the description of classes of quantum correlations and in the introduction and study of quantum graph parameters, including quantum chromatic numbers and projective ranks. Their connection with Tsirelson's Problem and Connes' Embedding Problem will be discussed, and the link between these problems and the introduced graph parameters will be highlighted.

Christophe Lacave (Université Paris-Diderot),

Incompressible fluids through a porous medium

Abstract. In a perforated domain, the asymptotic behavior of the fluid motion depends on the rate (inter-hole distance)/(size of the holes). We will present the standard framework and explain how to find the critical rate where "stange terms" appear. Next, we will compare the critical rate for the Laplace, Navier-Stokes and Euler equations.

Gustav Holzegel (Imperial College London),

Linear Stability of the Schwarzschild solution under gravitational perturbations

Abstract. The well-known Schwarzschild solution is a spherically symmetric static solution of the vacuum Einstein equations describing a black hole. In my talk, I will outline a recent proof, obtained in collaboration with

Dafermos and Rodnianski, of the linear stability of the Schwarzschild solution under gravitational perturbations. The proof combines insights on the behaviour of linear waves on black hole backgrounds (proven in the last ten years) with a hierarchical structure in the system of linearised Einstein equations.

Session 31, December 11, 2015 in Paris (Institut Henri-Poincaré)

Mariapia Palombaro (University of Sussex),
Higher gradient integrability for two-phase conductivities in dimension two.

Abstract. I will present some results concerning the higher gradient integrability of σ-harmonic functions u with discontinuous coefficients σ, i.e. weak solutions of $\mathrm{div}(\sigma\nabla u) = 0$. When σ is assumed to be symmetric, then the optimal integrability exponent of the gradient field is known thanks to the work of Astala and Leonetti & Nesi. I will discuss the case when only the ellipticity is fixed and σ is otherwise unconstrained and show that the optimal exponent is attained on the class of two-phase conductivities: $\sigma : \Omega \subset \mathbb{R}^2 \mapsto \{\sigma_1, \sigma_2\} \subset \mathbb{M}^{2\times 2}$. The optimal exponent is established, in the strongest possible way of the existence of so-called exact solutions, via the exhibition of optimal microgeometries. (Joint work with V. Nesi and M. Ponsiglione.)

Benoît Grebert (Université de Nantes),
On reducibility of quantum harmonic oscillator on $\mathbb{R}^d$ with quasi periodic in time potential.

Abstract. We prove that a linear d-dimensional Schrödinger equation on $\mathbb{R}^d$ with harmonic potential x^2 and small t-quasiperiodic potential

$$i\partial_t u = -\partial_x^2 u + |x|^2 u + \varepsilon V(t\omega, x)u, \quad x \in \mathbb{R}^d$$

reduces to an autonomous system for most values of the frequency vector ω. As a consequence any solution of such a linear PDE remains bounded in all Sobolev norms.

Claude Warnick (Imperial College, London),
Scattering resonances and black holes.

Abstract. Many types of black hole respond to linear perturbations by ringing like a bell. The associated characteristic frequencies are complex, representing behaviour that is both oscillatory and decaying. I will present recent work establishing rigorously the properties of the spectrum for a large class of black holes. I will connect the problem to the study of the meromorphicity of the resolvent for asymptotically hyperbolic manifolds, and show that the methods introduced for the black hole problem give a novel proof of a classical result of Mazzeo-Melrose.)

Frédéric Rousset (Université Paris 11 - Orsay),
Quasineutral limit for Vlasov-Poisson systems.

Abstract. We shall discuss the quasi-neutral limit of the Vlasov Poisson system in Sobolev spaces. We will prove in particular the local well-posedness of the limit system, which is a Vlasov type equation with Dirac interaction potential, for initial data in Sobolev spaces for which the profile in the velocity variable satisfies a stability condition. (Joint work with D. Han-Kwan)

Session 30, October 23, 2015 in London (Imperial College)

Spyros Alexakis (University of Toronto, Canada),
Control of wave equations from data on time-like surfaces, and applications to the profile of singularities

Abstract. We review some recent results, joint with A. Shao (and partly Volker Schlue), regarding the control one can obtain for some linear and non-linear wave equations, from Cauchy data on suitably large portions of time-like surfaces. We obtain control of the solution on specific space-like surfaces which have the property that any null ray crossing the surface "registers" on the Cauchy data set. This is applied to understand the singularity profile of some focusing non-linear waves.

Camille Laurent (CNRS, Université Pierre-et-Marie-Curie, Paris),
Quantitative unique continuation for operators with partially analytic coefficients. Application to approximate control for waves.

Abstract. Unique continuation is very often proved by Carleman estimates or Holmgren theorem. The first one requires the strong geometric assumption of pseudoconvexity of the hypersurface. The second one only requires

that the hypersurface is non characteristic, but the coefficients need to be analytic.
Motivated by the example of the wave equation, several authors (Tataru, Robbiano-Zuily, Hörmander) finally proved in great generality that there could be unique continuation in some intermediate situation where the coefficients are analytic in part of the variables. In particular, for the wave equation, it allowed to prove the unique continuation across any non characteristic hypersurface for non analytic metric.
In this talk, after presenting these works, I will describe some recent work where we quantify this unique continuation. This leads to the optimal (in general) logarithmic stability estimates. We will also give some applications to controllability. This is joint work with Matthieu Léautaud (Université Paris-Diderot).

José Rodrigo (University of Warwick),
On non-resistive MHD systems connected to magnetic relaxation.

Abstract. In this talk I will present several results connected with the idea of magnetic relaxation for MHD, including some new commutator estimates (and a counterexample to the estimate in the critical case). (Joint work with various subsets of D. McCormick, J. Robinson, C. Fefferman and J-Y. Chemin.)

Roman Novikov (CNRS, École Polytechnique, Palaiseau),
Inverse scattering at fixed energy with non-overdetermined data.

Abstract. We consider the problem of reconstruction of the potential in the Schrödinger equation from the scattering amplitude at a fixed energy in dimension $d = 2, 3, \ldots$. The main purpose of this talk consists in consideration of this problem in non-overdetermined formulation, that is when the scattering amplitude at fixed energy is given on appropriate d-dimensional sub-manifolds of its domain of definition. The main attention is paid to the three-dimensional case: $d = 3$. Our results include, in particular, the first efficient approximate reconstruction algorithm and related stability estimates for the non-overdetermined three-dimensional inverse scattering problem at sufficiently high fixed energy.

Session 29, June 19, 2015 in Paris (Institut Henri-Poincaré)

held within a 3 month-program on inverse problems

Daniel Tataru (UC Berkeley, USA),
The energy critical Maxwell Klein Gordon evolution.

Abstract. The Maxwell Klein Gordon fits into the class of geometric nonlinear wave equations, and is closely related to wave maps and the hyperbolic Yang-Mills system. In 4+1 dimensions this problem is energy critical. I will describe recent work, joint with Joachim Krieger and Jacob Sterbenz (for small data) and with Sung-Jin Oh (for large data), on global well-posedness for this problem. In particular, I will discuss the paradifferential gauge renormalization that is crucial to all these results.

Keith Rogers (ICMAT, Madrid, Spain),
Global uniqueness for the Calderón problem with Lipschitz conductivities.

Abstract. We will review recent progress for Calderón's inverse problem in which one hopes to determine the conductivity γ of a body $\Omega \subset \mathbb{R}^n$. In order to do this, voltages are placed on the boundary $\partial\Omega$, and the induced currents, perpendicular to $\partial\Omega$, are measured. In other words, we hope to recover the conductivity from the Dirichlet-to-Neumann-map of the associated conductivity equation. With $n \geq 3$, we prove that no two Lipschitz conductivities give rise to the same Dirichlet-to-Neumann-map, extending a recent result of Haberman, who proved uniqueness for conductivities in the larger class $L^\infty \cap W^{1,n}$ with $n = 3$ or 4. Our proof builds on the work of Sylvester and Uhlmann, Brown, and Haberman and Tataru who proved uniqueness for C^1-conductivities and Lipschitz conductivities sufficiently close to the identity (as long as $\|\nabla \log \gamma\|_\infty$ is sufficiently small). We will recall their ideas, before sketching the proof of a Carleman estimate that we use in order to remove the smallness condition. This is joint work with Pedro Caro.

Maarten de Hoop (Purdue University, USA),
Title TBA

Abstract.

Adrian Nachman (University of Toronto, Canada),
Imaging Conductivity from one Internal Current Measurement and Minimal Surfaces.

Abstract. This talk will give an overview of electric conductivity imaging from interior data obtainable using Magnetic Resonance Imagers, and the beautiful underlying Riemannian structure. We show that an anisotropic conductivity in a known conformal class can be determined from measurement of one current using geometric measure theory methods. Further, we show that the associated equipotential surfaces are area minimizing with respect to a Riemannian metric obtained entirely from the physical data. We treat both Dirichlet boundary conditions, as well as those coming from the Complete Electrode Model, which we will descibe.
This describes results obtained in several joint papers with Nicholas Hoell, Robert Jerrard, Amir Moradifam, Alexandru Tamasan and Johann Veras. The experimental results are joint work with Weijing Ma, Nahla Elsaid, Michael Joy and Tim DeMonte.

Session 28, March 27, 2015 in London (King's College London)

Laurent Michel (Université de Nice-Sophia Antipolis),
Tunnel effect for a semiclassical random walk.

We consider a semiclassical random walk associated to a probability density with a finite number of wells. We study the spectrum of the associated Markov operator and give an asymptotics of the highest eigenvalues. The key ingredient in our approach is a general factorization result of pseudodifferential operators, which allows us to use recent results of the Witten Laplacian. This is a joint work with J.-F. Bony and F. Hérau.

Svetlana Jitomirskaya (Isaac Newton Institute, Cambridge, UK and UCI, USA),
Quasiperiodic Operators with Monotone Potentials: Sharp Arithmetic Spectral Transitions and Small Coupling Localization.

It is well known that spectral properties of quasiperiodic operators depend rather delicately on the arithmetics of the parameters involved. Consequently, obtaining results for all parameters often requires considerably more difficult arguments than for a.e. parameter, and does offer a deeper insight. In the first part of the talk we will report the first result of this kind in regard to the spectral decomposition: full description of spectral types of the Maryland model for all (in contrast with a.e., known for 30 years) values of frequency, phase, and coupling with nontrivial dependence on the arithmetics (joint work with W. Liu). In the second part of the talk we show that for (a large class of) bounded monotone potentials there is Anderson localization for all non-zero couplings (joint work with I. Kachkovskiy).

Dimitri Yafaev (Université de Rennes),
Spectral and scattering theory for differential and Hankel operators

We consider a class of Hankel operators H realized in the space $L^2(\mathbb{R}_+)$ as integral operators with kernels $h(t+s)$ where $h(t) = P(\ln t)t^{-1}$ and $P(X) = X^n + p_{n-1}X^{n-1} + \cdots$ is an arbitrary real polynomial of degree n. This class contains the classical Carleman operator when $n = 0$. We show that Hankel operators H in this class can be reduced by an *explicit* unitary transformation (essentially by the Mellin transform) to a differential operator $A = vQ(D)v$ in the space $L^2(\mathbb{R})$. Here $Q(X) = X^n + q_{n-1}X^{n-1} + \cdots$ is a polynomial determined by $P(X)$ and $v(\xi) = \pi^{1/2}\big(\cosh(\pi\xi)\big)^{-1/2}$ is the universal function. Then the operator $A = vQ(D)v$ reduces by the generalized Liouville transform to the standard differential operator $B = D^n + b_{n-1}D^{n-1} + \cdots + b_0(x)$ with the coefficients $b_m(x)$, $m = 0, \ldots, n-1$, decaying sufficiently rapidly as $|x| \to \infty$. This allows us to use the results of spectral theory of differential operators for the study of spectral properties of generalized Carleman operators. In particular, we show that the absolutely continuous spectrum of H is simple and coincides with $\mathbb{R}$ if n is odd, and it has multiplicity 2 and coincides with $[0, \infty)$ if $n \geq 2$ is even. The singular continuous spectrum of H is empty, and its eigenvalues may accumulate to the point 0 only. As a by-product of our considerations, we develop spectral theory of differential operators $A = vQ(D)v$ with sufficiently arbitrary functions $v(\xi)$ decaying at infinity.

Yulia Karpeshina (University of Alabama at Birmingham and Isaac Newton Institute, Cambridge),
Absolutely continuous branch of the spectrum and quantum transport properties of Schrödinger operator with a limit-periodic potential in dimension two

Existence of absolutely continuous branch of the spectrum and ballistic transport for Schrödinger operator with a limit-periodic potential in dimension two is discussed. Considerations are based on the following properties of the operator: the spectrum of the operator contains a semiaxis and there are generalized eigenfunctions being close to plane waves $e^{i\langle \vec{k}, \vec{x}\rangle}$ (as $|\vec{k}| \to \infty$) at every point of this semiaxis. The isoenergetic curves in the space of momenta $\vec{k}$ corresponding to these eigenfunctions have a form of slightly distorted circles with holes (Cantor type structure).

Session 27, December 12, 2014 in Paris (Institut Henri-Poincaré)

Tadahiro Oh (The University of Edinburgh),
Invariant Gibbs measures for the nonlinear Schrödinger equations on the circle and the real line.
In this talk, we first go over the construction of invariant Gibbs measures for the nonlinear Schrödinger equations (NLS) on the circle by Bourgain '94. Then, we discuss the situation on the real line by taking larger and larger periods. In particular, we realize the limiting Gibbs measure on the real line as a diffusion process in x and prove its invariance for (sub-)quintic NLS on the real line.

Hajer Bahouri (CNRS, Université Paris-Est),
On nonlinear Schrödinger equations with exponential growth.
We describe the feature of solutions to nonlinear Schrödinger equations with exponential growth, where the Orlicz norm plays a crucial role. Based on profile decompositions, the analysis we conducted in this work emphasizes that the nonlinear effect is only generated by the **1**-oscillating component of the sequence of the Cauchy data. This phenomenon is strikingly different from those observed in scale invariant equations, where all the oscillating components have the same impact on the behavior of the solutions. One of the key arguments of our approach relies on refined Strichartz estimates involved Bourgain spaces.

Jonathan Luk (University of Cambridge),
Singularities in general relativity
I will present some recent developments in constructing stable low-regularity solutions to the Einstein equations without any symmetry assumptions. These in particular include stable spacetimes which contain singularities propagating along null hypersurfaces. I will discuss some applications of these techniques for various problems in general relativity, including understanding the interaction of impulsive gravitational waves, the formation of trapped surfaces and the singularity structure in the interior of black holes. This talk is based on results obtained in collaboration with M. Dafermos

and I. Rodnianski.

Jérémie Szeftel (CNRS, Université Pierre-et-Marie-Curie),
The resolution of the bounded L^2 curvature conjecture in general relativity.
In order to control locally a space-time which satisfies the Einstein equations, what are the minimal assumptions one should make on its curvature tensor? The bounded L^2 curvature conjecture roughly asserts that one should only need L^2 bounds of the curvature tensor on a given space-like hypersurface. This conjecture has its roots in the remarkable developments of the last twenty years centered around the issue of optimal well-posedness for nonlinear wave equations. In this context, a corresponding conjecture for nonlinear wave equations cannot hold, unless the nonlinearity has a very special nonlinear structure. I will present the proof of this conjecture, which sheds light on the specific null structure of the Einstein equations. This is joint work with Sergiu Klainerman and Igor Rodnianski.

Session 26, October 10, 2014 in London (University College London)

Claude-Alain Pillet (Centre de Physique Théorique, Université de Toulon),
The Landauer Principle in quantum statistical mechanics.
In a celebrated 1961 paper, Landauer formulated a fundamental lower bound on the energy dissipated by computation processes. Since then, there has been many attempts to formalize, generalize and contradict Landauer's analysis. The situation became even worse with the advent of quantum computing. In a recent enlightening article, Reeb and Wolf sets the game into the framework of quantum statistical mechanics, and finally gave a precise mathematical formulation of Landauer's bound. I will discuss parts of this analysis and present some extensions of it that were obtained in a joint work with V. Jaksic.

Michiel van den Berg (University of Bristol),
Minimization of Dirichlet eigenvalues.
We discuss the problem of minimizing the k'th eigenvalue of the Dirichlet Laplacian over all open sets which satisfy one or more geometric constraints for example (i) both Lebesgue measure and perimeter bounded from above, or (ii) moment of inertia bounded from above and convexity.

Gérard Besson (Université Joseph Fourier, Grenoble),
On some open 3-manifolds
We describe some open 3-manifolds whose Riemannian Geometry is widely unknown. Most of them are submanifolds of the 3-sphere, complement of Cantor sets or fractals embedded in S3. We will survey some open questions both in analysis and in geometry. Some of these questions may even be thought of as pertaining to Geometric inverse problems.

Kirill Cherednichenko (Cardiff University),
Resolvent estimates for high-contrast elliptic problems with periodic coefficients.
I will discuss the asymptotic behaviour of the resolvents $(\mathcal{A}^\varepsilon + I)^{-1}$ of elliptic second-order differential operators $\mathcal{A}^\varepsilon$ in $\mathbb{R}^d$ with periodic rapidly oscillating coefficients, as the period ε goes to zero. The class of operators covered by the discussion includes both the "classical" case of uniformly elliptic families (where the ellipticity constant does not depend on ε) and the "double-porosity" case of coefficients that take contrasting values of order one and of order ε^2 in different parts of the period cell. I shall describe a construction for the leading order term of the "operator asymptotics" of $(\mathcal{A}^\varepsilon + I)^{-1}$ in the sense of operator-norm convergence and prove order $O(\varepsilon)$ remainder estimates. This is joint work with Shane Cooper.

Session 25, June 20, 2014 in Paris (Institut Henri-Poincaré)

Karine Beauchard (École Polytechnique),
Controllability of degenerate parabolic equations: minimal time and geometric control condition.
We consider degenerate parabolic equations of Grushin type and of Kolmogorov type, on rectangle domains. We study their null controllability in the usual L^2 setting. The control is a source term localized on an open subset of the rectangle. We will see that, depending on the strength of the degeneracy, the form of the control support, and the time allowed, this controllability property holds or does not hold. These fact contrasts with the classical results proved in the uniformly parabolic case (heat equation).

Gui-Qiang G. Chen (Oxford University),
Multidimensional Transonic Shocks and Free Boundary Problems.

In this talk, we shall analyze several longstanding, fundamental multidimensional shock problems in mathematical fluid mechanics and related free boundary problems for nonlinear partial differential equations of mixed elliptic-hyperbolic type. These shock problems include supersonic flow onto a solid wedge (Prandtl-Meyers problem), shock reflection-diffraction by a concave cornered wedge (von Neumann's conjectures), and shock diffraction by a convex cornered wedge (Lighthills problem). Some recent developments and related mathematical challenges in solving these problems will be discussed. Further trends and open problems in this direction will also be addressed.

Eugen Varvaruca (University of Reading),
Singularities of steady free surface water flows under gravity.
We shall present some recent results which provide a characterization, by means of geometric methods, of all possible singularities in two related free-boundary problems in hydrodynamics: that of steady two-dimensional gravity water waves and that of steady three-dimensional axisymmetric water flows under gravity. In the 2D problem, we shall outline a modern proof, using blow-up analysis based on a monotonicity formula and a frequency formula, of the famous Stokes conjecture from 1880, which asserts that at any stagnation point on the free surface of a steady irrotational gravity water wave, the wave profile necessarily has lateral tangents enclosing a symmetric angle of 120 degrees. This result was first proved in the 1980s under some restrictive assumptions and by somewhat ad-hoc methods. The new approach extends to the case when the effects of vorticity in the flow are included. Moreover, we shall explain how the methods can be adapted to the 3D axisymmetric problem, which exhibits a much richer behaviour as far as singularities are concerned, depending on whether one is dealing with a stagnation point, a point on the axis of symmetry, or both (in the case of the origin). For example, in the case of the origin, there are two possible types of singular asymptotic behaviour: one is a conical singularity called "Garabedian corner flow", and the other is a flat degenerate point; while in the case of points on the axis of symmetry different from the origin, cusps are the only possible singularities. These results were obtained in joint works with Georg Weiss (Dusseldorf).

Christophe Cheverry (Université de Rennes 1),
Can one hear whistler waves ?

In this talk, we will present two results. The first provides a new approach allowing to extend in longer times the classical insights on fast rotating fluids. This will be applied to show that a plasma can be confined by a magnetic field. The second is based on a study of oscillatory integrals implying special phases. This will be applied to give a better understanding of whistler-mode chorus emissions in space plasmas. The framework will be relativistic Vlasov-Maxwell equations, with a penalized skew-symmetric term where the inhomogeneity of the magnetic field plays an essential part.

Session 24, March 28, 2014 in London (Queen Mary University)

Yaroslav Kurylev (University College London),
Discrete metric measure approximation and spectral convergence for Riemannian manifolds.
We consider a discrete ε-metric-measure approximation (X, d, μ) to a compact Riemannian manifold (M, g) of bounded sectional curvature and injectivity radius. We define a ρ-Laplacian, $\Delta_{\varepsilon,\rho}$ on X, $\varepsilon \ll \rho \ll 1$, and analyse the spectral convergence of $\Delta_{\varepsilon,\rho}$ to Δ_M. This is a joint work with D. Burago (PennState Univ, USA) and S. Ivanov (PDMI, Russia).

Frank Merle (Université de Cergy-Pontoise & IHES),
Blow up for mass critical KdV and universality properties near the solitary wave.
We shall give a review of old and recent results on mass critical KdV related to blow-up. A classification of all possible dynamics around the solitary wave will be given and some related problems. Joint work with Martel and Raphael.

Didier Robert (Université de Nantes),
On random Hermite series and applications.
In this lecture, we shall present some smoothing properties obtained for Hermite expansions with random coefficients in $L^2(\mathbb{R}^d)$, extending in this case known classical results for Fourier series, like the Paley-Zygmund and Salem-Zygmund theorems. In particular we get a random Strichartz inequality which can be applied to discuss local and global well-posedeness for super-critical non-linear Schrödinger equations, with and without harmonic potentials, for random initial data. We will survey these results obtained in

several joint works with Rafik Imekraz, A. Poiret, and L. Thomann.

Tom Körner (Cambridge University),
Convolution Squares.
When we convolve two functions, the result is sometimes smoother than we expect. The talk investigates the degree to which this can happen.

Session 23, December 13, 2013 in Paris (Institut Henri-Poincaré)

Pascal Auscher (Université Paris Sud),
A new proof of Koch-Tataru result for Navier-Stokes equation with BMO^{-1} data.
This result bears on the boundedness of a bilinear operator on a solution space using parabolic Carleson functions. The space of such functions is a sample of a (parabolic) tent space as defined by Coifman, Meyer, Stein. We shall explain the more systematic use of tent space in our approach of the bilinear operator. In particular, we do not rely on self-adjointness of the Laplacian. This is joint work with Dorothee Frey (ANU).

Charles Batty (University of Oxford),
Rates of decay in Tauberian theorems.
Ingham proved in 1935 that if $f \in L^\infty(0,\infty)$ and its Laplace transform $\hat{f}$ has a holomorphic extension across the imaginary axis, then the improper integral of f over $(0,\infty)$ exists and equals $\hat{f}(0)$. New proofs emerged in the 1980s as parts of elementary proofs of the Prime Number Theorem. This led to a qualitative theorem about decay of smooth orbits of operator semigroups. Recent research into rates of decay of energy in damped wave equations has inspired further investigation of the rates of converegence in these results and to L^p-versions of Ingham's theorem. I will survey these results including recent joint work with Yuri Tomilov, Ralph Chill and Alexander Borichev.

Nicolas Rougerie (CNRS & Université Joseph Fourier),
Classical Coulomb gases beyond mean-field theory.
Classical Coumb systems are fundamental models of matter and have a well-known relevance to the study of random matrices ensembles, the Ginzburg-Landau theory of superconductivity, and the fractional quantum Hall effect.

In this talk, we will consider a large system of N classical charged particles interacting via Coulomb forces in space dimension d=2 or larger, trapped in a confining electrostatic potential. Assuming that the strength of the interaction scales as the inverse of N (mean-field regime), it is well-known that the leading order of the ground state energy is given by a mean-field (continuum) theory. We will be interested in going to the next order and investigate the fluctuations around mean-field theory. We prove that these are described by the minimization of a "renormalized energy" functional that gives the energy per unit volume of infinitely many charged particles interacting with each other and with a constant neutralizing background of opposite charge (infinite jellium). Exploiting coercivity properties of this functional, we deduce estimates on the precision of mean-field theory, both at zero and positive temperature. This is a joint work with Sylvia Serfaty.

Véronique Fischer (Imperial College London),

Pseudo-Differential Operators on nilpotent Lie groups.

The aim of this talk is to present recent developments in pseudo-differential calculi on nilpotent Lie groups as well as some historical motivations. This is a joint work with Professor Michael Ruzhansky (Imperial College London).

Session 22, October 11, 2013 in London (University College)

Dmitry Jakobson (McGill University),

Averaging over Riemannian metrics.

I will survey several recent results related to averaging over different spaces of Riemannian metrics. The first result is joint work with Y. Canzani and J. Toth. We study the moments of propagated perturbed eigenfunctions, evaluated at a fixed point x on a compact manifold, considered as random variables that arise from random perturbations of a metric. The (finite-dimensional) family of Schrödinger operators corresponds to perturbations of the reference Riemannian metrics. Assuming the perturbation family has nontrivial projections onto conformal changes of the metric at x, we establish asymptotics for the odd moments. Assuming the perturbation family has non-degenerate projection onto the space of volume-preserving transformations at x, we establish bounds for the variance. The perturbed eigenfunctions arise in the study of Loschmidt echo effect in physics. If time permits, I will speak about the second result (joint work with Y. Canzani, B. Clarke, N. Kamran, L. Silberman and J. Taylor). We define Gaussian measures on manifolds of metrics with the fixed volume form. We next prove integrability results for diameter and Laplace eigenvalue functionals of the random Riemannian metric.

Michel Pierre (ENS Cachan, Ker Lann),
An introduction to shape optimization: some mathematical issues.
Shape optimization is a specific part of calculus of variations where the variable is a geometric object, often called “a shape”, which is most of the time a subset of the plane, of the space or of a d-dimensional space. Shape optimization is a very old topic, but recent years have seen renewed interest for its study, due to the countless underlying applications and to the challenging mathematical questions behind. In this survey talk, we will present some mathematical tools and results related to shape optimization. We will address for instance the questions of existence of optimal shapes, of shape differentiation and of regularity of optimal shapes (together with open problems).

Jim Wright (University of Edinburgh),
A calculus for oscillatory integrals ?
Starting with a uniform oscillatory integral bound with a given phase, we ponder the possibility of deducing uniform oscillatory integral estimates for polynomial changes of the phase. Our investigations lead to some elementary considerations of how certain geometric quantities associated to the roots of a polynomial (for example, the diameter of the roots) depend on the coefficients of the polynomial.

Matthieu Léautaud (Université Paris 7),
Damped wave equation on the torus.
We address the decay rates of the energy for the damped wave equation on the torus. When the damping coefficient does not satisfy the Geometric Control Condition (i.e. in the presence of “trapped rays”), the decay is known to fail to be exponential. In such situations, we first prove that the decay is always polynomial and then investigate the optimal polynomial rate. We prove that the relevant feature in this study is the rate at which the damping coefficient vanishes. We finally discuss different situations in which the damping coefficient is invariant in one direction. The methods used include resolvent estimates and second microlocalizations around trapped

rays. This is based on joint works with Nalini Anantharaman and Nicolas Lerner.

Session 21, June 28, 2013 in Paris (Institut Henri-Poincaré)

Christian Lubich (Universität Tübingen),
Modulated Fourier expansions for continuous and discrete oscillatory systems.
This talk reviews some of the phenomena and theoretical results on the long-time energy behaviour of continuous and discretized oscillatory systems that can be explained by modulated Fourier expansions: long-time preservation of total and oscillatory energies in oscillatory Hamiltonian systems and their numerical discretisations, near-conservation of energy and angular momentum of symmetric multistep methods for celestial mechanics, metastable energy strata in nonlinear wave equations, and long-time stability of plane wave solutions of nonlinear Schrödinger equations. We describe what modulated Fourier expansions are and what they are good for. Most of the presented work was done in collaboration with Ernst Hairer. Some of the results on modulated Fourier expansions were obtained jointly with David Cohen and Ludwig Gauckler.

Horia Cornean (Aalborg University),
On the steady state correlation functions of open interacting systems.
We address the existence of steady state Green-Keldysh correlation functions of interacting fermions in mesoscopic systems for both the partitioning and partition-free scenarios. Under some spectral assumptions on the non-interacting model and for sufficiently small interaction strength, we show that the system evolves to a NESS which does not depend on the profile of the time-dependent coupling strength/bias. For the partitioned setting we also show that the steady state is independent of the initial state of the inner sample. This is joint work with V. Moldoveanu (Bucharest) and C.-A. Pillet (Toulon).

Annalisa Panati (Université du Sud-Toulon-Var),
Entropic fluctuations in non-equilibrium statistical mechanics for spin-boson systems.
We consider a finite level quantum system interaction with many (bosonic) heat reservoir. Using methods of spectral analysis of Liouvellean opera-

tors, we study fluctuation of the entropy fluxes (central limit theorem, large deviation principle). This is a joint work with V. Jaksic, C.A. Pillet, M. Westrich.

Jean Dolbeault (Université Paris Dauphine, CNRS),
Rigidity results, inequalities and nonlinear flows on compact manifolds.

Session 20, March 22, 2013 in London (Imperial College)

Gunther Uhlmann (University of Washington, Fondation Sciences Mathématiques de Paris),
Multiwave Imaging.
Multi-wave imaging methods, also called hybrid methods, attempt to combine the high resolution of one imaging method with the high contrast capabilities of another through a physical principle. One important medical imaging application is breast cancer detection. Ultrasound provides a high (sub-millimeter) resolution, but suffers from low contrast. On the other hand, many tumors absorb much more energy of electromagnetic waves (in some specific energy bands) than healthy cells. Photoacoustic tomography (PAT) consists of sending relatively harmless optical radiation into tissues that causes heating which results in the generation of propagating ultrasound waves (the photo-acoustic effect). Such ultrasonic waves are readily measurable. The inverse problem then consists of reconstructing the optical properties of the tissue from these measurements. In Thermoacoustic tomography (TAT) low frequency microwaves, with wavelengths on the order of $1m$, are sent into the medium. The rationale for using the latter frequencies is that they are less absorbed than optical frequencies. Transient Elastography (TE) images the propagation of shear waves using ultrasound. Multi-wave imaging methods lead to a rich supply of new mathematical questions that involve elliptic and hyperbolic partial differential equations. We will discuss some of the inverse problems arising in these imaging techniques with emphasis on PAT.

Jonathan R. Partington (University of Leeds),
Near invariance and kernels of Toeplitz operators.
This talk presents a study of kernels of Toeplitz operators on scalar and vector-valued Hardy spaces. The property of near invariance of a kernel

for the backward shift is shown to hold in much greater generality. In the scalar case, and in some vectorial cases, the existence of a minimal kernel containing a given function is established, and a corresponding Toeplitz symbol is determined; thus for rational symbols its dimension can be easily calculated. It is shown that every Toeplitz kernel is the minimal kernel for some function lying in it. This is joint work with Cristina Camara (Lisbon).

Pierre Degond (CNRS, Université Paul Sabatier, Toulouse),
Phase transition, hysteresis and hydrodynamic limit in models of self propelled particles interacting through local alignment.
Systems of self-propelled particles can be observed in nature at a wide variety of scales from swarming bacteria, and collectively moving cells to insect swarms, bird flocks and fish schools. In 1995, Vicsek and co-authors proposed a paradigmatic model for these phenomena, where particles tend to locally align to the average direction of their neighbors. They observed phase transitions from disordered motion to coherent collective motion when the density exceeds a certain threshold. A controversy arose in the physics community about the order of this phase transition. The goal of this talk is to provide a mathematically rigorous perspective to this discussion in a kinetic theory formalism. We prove that, according to the interaction rules, the transition can be either second-order (continuous) or first-order (discontinuous), the latter giving rise to hysteresis phenomena. We then discuss how hydrodynamic-like models for self-propelled particle systems can be constructed and show that these models present specific features that standard hydrodynamic models do not have.

Marco Marletta (Cardiff University),
On the stability of a forward-backward heat equation.
We examine the spectral properties of a family of periodic singular Sturm-Liouville problems which are highly non-self-adjoint but have purely real spectrum. The problem originated from the study of the lubrication approximation of a viscous fluid film in the inner surface of a rotating cylinder and has received substantial attention in recent years. We determine the Schatten class inclusions for the resolvent operator and properties of the associated evolution equation. This is joint work with Lyonell Boulton and David Rule.

Session 19, December 7, 2012 in Paris (Institut Henri-Poincaré)

Francis Nier (Université de Rennes 1 & INRIA),

About the method of characteristics.

While studying the mean field dynamics of a systems of bosons, one is led to solve a transport equation for a probability measure in an infinite dimensional phase-space. Those probability measures are characterized after testing with cylindrical or polynomial observables classes which are not invariant after composing with a nonlinear flow. Thus, the standard method of characteristics for transport equations cannot be extended at once to the infinite dimensional case. A solution comes from techniques developed for optimal transport and a probabilistic interpretation of trajectories. This is extracted from a joint work with Z. Ammari.

Igor Krasovsky (Imperial College London),

Double-scaling asymptotics for Toeplitz determinants.

We will discuss double-scaling asymptotics of Toeplitz determinants that display a "phase-transition". Close to the transition point the asymptotics are given in terms of Painleve functions. A typical example is the 2-spin correlation function in the 2D Ising model at the critical temperature. We will provide this and some other examples. The talk is based in part on the joint work with A.Its and T.Claeys.

Boguslaw Zegarlinski (Imperial College London),

Generalized gradient bounds and applications.

We shall discuss generalized gradient bounds for Markov semigroups and some applications including construction and ergodicity in a finite and infinite dimensional context.

Taoufik Hmidi (Université de Rennes 1),

On the regularity of the rotating vortex patches.

In this talk we discuss some special vortex patches for the two-dimensional incompressible Euler equations which preserve their shape during the motion. The simplest examples are given by Rankine and Kirchhoff vortices which are subjected to a uniform rotation around their centers. We know from the works of Deem-Zabusky and Burbea that there is a general class of rotating vortex patches, called the V-states and bifurcating from the circle at the eigenvalues of a certain linearized operator. We will show that the

V-states are convex and C^∞ close to the circle. The lecture is based on a joint work with Mateu and Verdera.

Session 18, October 5, 2012 in London (University College)

Frédéric Hérau (Université de Nantes),
Subelliptic estimates for the inhomogeneous Boltzmann equation without cut-off.
We provide global subelliptic estimates for the Bolzmann equation without angular cutoff, and show that some global gain in the spatial direction is available although the corresponding operator is not elliptic in this direction. Due to the bad symbolic properties of the operator (in the microlocal sense), the proof uses the so-called Wick quantization and some ideas coming from semi-classical analysis. This is a joint work with W.-X. Li (Wuhan) and R. Alexandre (Shanghai and Brest).

Alexander Strohmaier (Loughborough University),
Semiclassical Analysis for Discontinous Systems and Ray-Splitting Billiards.
Many questions in spectral theory are motivated by Bohr's correspondence principle which states that in the semi-classical limit classical mechanics can be recovered from Quantum mechanics. For spectral theory of the Laplace operator on compact manifolds that means that in the limit of high energy the geodesic flow determines the behaviour of the spectrum and that of eigenfunctions. A precise version of this is Egorov's theorem that links the quantum dynamics explicitly with the geodesic flow via the symbol map for pseudodifferential operators. Another classical theorem is the quantum ergodicity theorem which states that most eigenfunctions become equidistributed for large eigenvalues if the geodesic flow is ergodic. If the manifold has a metric with a jump-like discontinuity across a codimension one hypersurface Egorov's theorem does in general not hold in its classical form and the geodesic flow may not be well defined any more: geodesics hitting the discontinuity may split into reflected and refracted rays. We will prove a quantum ergodicity theorem for such manifolds relating the ergodicity of a ray-splitting dynamics to equidistribution of eigenfunctions. This is a joint work with D. Jakobson and Y. Safarov.

Stéphane Mallat (École Normale Supérieure),

From Fourier to Wavelet Scattering for Signal Classification.
Fourier analysis is powerful to characterize stationary properties and build translation invariant functional representations. Applications to signal and image processing are well known. However, a Fourier transform has high frequency instabilities under the action of diffeomorphisms. It thus becomes inappropriate to analyze the properties of functions that undergo complex deformations. As a result, it fails to characterize signal properties in most signal classification problems. Facing this issue, computer scientists have developed a jungle of new non-linear classification algorithms that go well beyond linear functional analysis. The seminar concentrates on this new area of non-linear functional analysis and its applications.
We prove that Lipschitz continuity to diffeomorphisms is obtained with a scale separation using a wavelet transform. A translation invariant and Lipschitz continuous functional representation results from a scattering decomposition which cascades wavelet transforms and non-linearities. This transform retains strong mathematical similarities with a Fourier integral, but it is Lipschitz continuous to diffeomorphisms. It is computed with a neural network architecture similar to algorithms used for classification. Several applications will be shown for image and audio texture classification, as well as hand-written digits and musical genre recognition.

Ben Green (Cambridge),
Approximate groups and applications.
We will give a survey on the topic of approximate groups. This is an area that has seen a lot of activity recently. Assuming no former knowledge, we will explain what an approximate group is, what is known about them, and some of the applications.

Session 17, June 22, 2012 in Paris (Institut Henri-Poincaré)

Maria Esteban (CNRS & Université Paris Dauphine),
Scenario for a symmetry breaking phenomenon in symmetric functional inequalities.
In this talk, I will present recent analytical and numerical work to understand symmetry breaking phenomena for optimizers in some functional inequalities, like for instance the Caffarelli-Kohn-Nirenberg inequalities. The unexpected symmetry breaking can be explained by a complicated bifurcation pattern for the branches of solutions of the corresponding Euler-

Lagrange equations.

André Martinez (Università di Bologna),
Padé approximants for the cubic oscillator (joint work with V. Grecchi).
We study the cubic oscillator Hamiltonian,

$$H(\beta) = -\frac{d^2}{dx^2} + x^2 + i\sqrt{\beta}x^3,$$

on $L^2(\mathbb{R})$, for β in the cut plane $\mathbb{C}_c$ consisting of all complex numbers that are not negative real numbers. We prove that the spectrum consists of simple eigenvalues only. Moreover, these eigenvalues depend analytically on $\beta \in \mathbb{C}_c$, are labeled by the constant number of nodes of the corresponding eigenfunction, and can be computed as the Stieltjes-Padé sum of their perturbation series at $\beta = 0$. This also gives an alternative proof of the fact that the spectrum of $H(\beta)$ is real when β is a positive number.

Adrian Constantin (King's College London),
Pressure beneath a traveling water wave.
Using harmonic function theory, we investigate the pressure within an irrotational fluid in a periodic, steady, two-dimensional gravity wave above a flat bed. We show that the pressure in the fluid strictly decreases horizontally away from the crest line. Furthermore, the pressure strictly increases with depth. The approach deals with the governing equations (incompressible Euler equations with a free boundary) and does not rely on approximations. In particular, it applies to waves of large amplitude. This is a joint work with W. Strauss.

David Gérard-Varet (Université Denis Diderot Paris 7),
Domain continuity for the Euler and Navier-Stokes equations.
The aim of the talk is to understand the effect of rough walls or rough obstacles on fluid flows. Mathematically, there are two natural ways to model the roughness:

1. by considering fluid domains with non-smooth boundaries.

2. by considering fluid domains with oscillating boundaries, the oscillation being of small amplitude and wavelength.

The first model often raises numerical and mathematical difficulties (like a lack of Cauchy theory), which requires to consider smooth approximations Ω^ε of the irregular domain Ω^0. As regards the second model, denoting by ε the small wavelength or amplitude of the oscillating boundary, one is also led to consider a sequence of domains Ω^ε parametrized by ε.

This leads naturally to questions of domain continuity for fluid models, broadly: if $\Omega\varepsilon$ converges to Ω^0, does the associated fluid velocity u^ε converge to u^0 ? Are the boundary conditions preserved in the limit ?

We shall investigate these questions in the context of the Euler and Navier-Stokes equations.

Session 16, March 23, 2012 in London (Queen Mary University)

D. Vassiliev (University College London),

The spectral function of a first order system.

We consider an elliptic self-adjoint first order pseudodifferential operator acting on columns of m complex-valued half-densities over a connected compact n-dimensional manifold without boundary. The eigenvalues of the principal symbol are assumed to be simple but no assumptions are made on their sign, so the operator is not necessarily semi-bounded. We study the spectral function, i.e. the sum of squares of Euclidean norms of eigenfunctions evaluated at a given point of the manifold, with summation carried out over all eigenvalues between zero and a positive λ. We derive a two-term asymptotic formula for the spectral function as λ tends to plus infinity. In doing this we establish that all previous publications on the subject give incorrect or incomplete formulae for the second asymptotic coefficient. We then restrict our study to the case when $m = 2$, $n = 3$, the operator is differential and has trace-free principal symbol, and address the question: is our operator a massless Dirac operator ? We prove that it is a massless Dirac operator if and only if the following two conditions are satisfied at every point of the manifold: a) the subprincipal symbol is proportional to the identity matrix and b) the second asymptotic coefficient of the spectral function is zero.

S. Nonnenmaher (CEA Saclay),

Chaotic damped waves.

The damped wave equation on a compact Riemannian manifold is a simple, yet rich nonselfadjoint problem. The study of its spectrum (made of complex eigenvalues in a strip below the real axis), in the high frequency/semiclassical limit, leads to a subtle interplay between the geodesic flow and the spatial distribution of the damping. We want to address the following question: which conditions ensure the presence of a spectral gap below the real axis, and hence the exponential decay of the wave energy ? We will mostly focus on cases where the geodesic flow (or at least some part of it) is hyperbolic, for instance if the manifold has negative sectional curvature.

J. Marklof (University of Bristol),

Eigenfunctions of rational polygons.

Consider an orthonormal basis of eigenfunctions of the Dirichlet Laplacian for a rational polygon. The modulus squared of the eigenfunctions defines a sequence of probability measures. I will show that this sequence contains a density-one subsequence that converges to Lebesgue measure. An important open problem is to classify all possible limit measures, and also to understand the corresponding microlocal lifts. The lecture is based on joint work with Zeev Rudnick.

A. Avila (Université Paris 7, IMPA),

Global theory of one-frequency Schrödinger operators.

One-frequency Schrödinger operators give one of the simplest models where fast transport and localization phenomena are possible. From a dynamical perspective, they can be studied in terms of certain one-parameter families of quasiperiodic cocycles, which are similarly distinguished as simplest classes of dynamical systems compatible with both KAM phenomena and nonuniform hyperbolicity (NUH). While being much studied since the 1970's, up to recently the analysis was mostly confined to "local theories" describing detailedly the KAM and the NUH regime. In this talk we will describe some of the main aspects of the global theory that has been developed in the previous few years.

Session 15, December 7, 2011 in Paris (Institut Henri-Poincaré)

Y. Capdeboscq,

Regularity Estimates in High Conductivity Homogenization.

In a recent work with Marc Briane and Luc Nguyen, we considered the case of a periodic micro-structure with highly conducting fibres (i.e. metal rods). Fenchenko and Khruslov showed 30 years ago that for a particular scaling

range, the effective problem includes a non-local term. We show that
(1) From a homogenization corrector result, one can deduce a lower bound on all norms $W^{1,p}_{loc}$ of the solution for $p > 2$, and this bound blows up like $\exp(C/\epsilon^2)$, a given power of the inverse of the radius of the rods.
(2) This is not a surface effect : the blow-up also occurs outside the fibres.
(3) Everywhere but at a distance less than $\epsilon^{1+\delta}$ from the fibres, the solution is uniformly $C^{1,\alpha}$ smooth. The measure of the forbidden domain tends to zero with a given rate in epsilon.
I will then discuss the interpretation of this result for two applications, a resolution problem in imaging, and meta-materials.

J.-Y. Chemin,
Blow-up condition for Navier-Stokes and Besov spaces with negative index.
In this talk, we want of extend the Escauriaza-Segerin-Sverak blow-up criteria in negative Besov spaces. Namely we want to prove that if u is a solution of the 3D incompressible Navier-Stokes equation and has a finite maximal time of existence T, then the norm in the Besov space $B^{-1+\frac{3}{p}}_{p,q}$ blows up near T for some p greater than 3 and for some q less than 2. The method consists in proving self-improving bounds on the solution which bypass the fact that the law of products is not valid for Besov spaces with negative indices.

I. Kachkovskiy,
Some results on almost-commuting operators.
A pair of almost-commuting operators is a pair of bounded self-adjoint operators with a small commutator. We are going to discuss various aspects of approximating such a pair with a pair of commuting operators, including some new results obtained jointly with N. Filonov.

F. Pacard,
Finite energy, sign changing solutions for the stationary non-linear Schrödinger equation.
I will explain how to construct finite energy solitary waves for nonlinear Klein-Gordon or Schrodinger equations. Under natural conditions on the nonlinearity, I will show the existence of finitely many nonradial solutions. In dimension 2, I will also explain how to construct finite energy solutions which have no symmetry at all.

Session 14, October 14, 2011 in London

J. Bennett,
Aspects of multilinear analysis related to Fourier restriction phenomena.
In this talk we will discuss the wide variety of geometric and combinatorial inequalities related to the restriction conjecture for the Fourier transform.

J.- M. Coron,
Controllability and stabilization of nonlinear control systems.
We present methods to study the controllability and the stabilizability of nonlinear control systems. The emphasis is put on specific phenomena due to the nonlinearities. In particular we study cases where the nonlinearities are essential for the controllability or the stabilizability. We illustrate these methods on specific control systems modeled by ordinary differential equations or partial differential equations.

B. Khoruzhenko,
Non-Hermitian Random Matrices.
In this talk I plan to survey statistical patterns in distribution of complex eigenvalues of real and complex random matrices. The emphasis will be on the exactly solvable ensembles, Gaussian and non-Gaussian, where one can derive the joint probability density function of eigenvalues induced by the matrix measure, and, consequently, the eigenvalue densities and higher-order correlations. The latter exhibit remarkable universality when scaled with the mean distance between eigenvalues. Proving such universality for a wide class of matrix distributions remains a challenging open problem.

C. Fermanian-Kammerer,
Coherent states and Nonlinear Schrödinger equation.
In this talk we will discuss the asymptotics of a family of solutions of a semi-classical nonlinear Schrödinger equation associated with initial data which are coherent states. More precisely, we will consider systems of such Schrödinger equations which are coupled by a matrix-valued potential. In this setting, we will describe nonlinear adiabatic theorems obtained recently.

Session 13, June 17, 2011 in Paris

T. Carbery,
The Multilinear Kakeya theorem, factorisation and algebraic topology.
We discuss some developments arising out of Guth's recent proof of the Multilinear Kakeya theorem which concern factorisation of functions, convex optimisation and aspects of algebraic topology.

S. Klainerman,
Rigidity of black holes.

A. Laptev,
Spectral inequalities for Dirichlet and Neumann Laplacians.
We discuss the properties of the eigenvalues of the Dirichlet and Neumann Laplacians on domains in the Euclidean space. In particular, we derive upper bounds on Riesz means that improve the sharp Berezin inequality by a negative second term. This remainder term depends on geometric properties of the boundary of the domain and reflects the correct order of growth in the semi-classical limit.

M. Rumin,
On some distribution-energy inequalities and related entropy bounds.
The talk will deal with some generalizations of inequalities by Li-Yau and Lieb-Thirring, that hold on general spaces and without Markovian or positivity assumption on the energy. They apply equally on functions or mixed states (density operators) and imply uncertainty principles involving various entropies associated to the states.

Session 12, March 4, 2011 in London

J. Robinson,
Numerical verification of regularity for solutions of the 3D Navier-Stokes equations.
I will show that one can (at least in theory) guarantee the "validity" of a numerical approximation of a solution of the 3D Navier-Stokes equations

using an explicit a posteriori test, despite the fact that the existence of a unique solution is not known for arbitrary initial data.
The argument relies on the fact that if a regular solution exists for some given initial condition, a regular solution also exists for nearby initial data ("robustness of regularity"); I will outline the proof of robustness of regularity for initial data in $H^{1/2}$.
I will also show how this can be used to prove that one can verify numerically (at least in theory) the following statement, for any fixed $R > 0$: every initial condition $u_0 \in H^1$ with $\|u\|_{H^1} \leq R$ gives rise to a solution of the unforced equation that remains regular for all $t \geq 0$.
This is based on joint work with Sergei Chernysehnko (Imperial), Peter Constantin (Chicago), Masoumeh Dashti (Warwick), Pedro Marń-Rubio (Seville), Witold Sadowski (Warsaw/Warwick), and Edriss Titi (UC Irvine-Weizmann).

S. Serfaty,
Derivation of a renormalized energy for Ginzburg-Landau vortex lattices.

P. Topping,
Regularity and compactness results for geometric PDE.
I will show how by understanding the geometry behind certain PDE, one can derive much better regularity and compactness properties for solutions than one might have initially expected. At the heart of the theory is the fact that Jacobian determinants have special properties in these regards, as has been exploited by various communities since the 1970s.
There will be no geometry prerequisites for this talk. I will do my best to make the talk appropriate both to experts and novices in the theory of PDE and harmonic analysis. Joint work with Ben Sharp.

M. Zworski,
Solitons in external fields.

Session 11, December 10, 2010 in Paris

M. Hillairet,

On a variational method to compute the forces exerted by a viscous incompressible fluid on a rigid body.
We consider a system coupling ODEs and Navier Stokes equations and modeling the motion of rigid bodies inside a viscous fluid. The Cauchy problem for this system is well-posed up to contact between two bodies or between one body and the boundary of the cavity (see [E. Feireisl, J. evol. equ. '03] and references therein). The 'contact problem' has been tackled by J.L. Vazquez and E. Zuazua (introducing a 1D toy-model), and by V.N. Starovoitov and T.I. Hesla independently. In a series of papers, we provide a new method for attacking this problem and give evidence that, when considering bodies with smooth boundaries, no contact is expectable in the 2D case, whereas it can occur in very specific 3D configurations. In this talk, I will explain this method and discuss its applications to numerical simulations and its extension to models including roughness of the rigid boundaries.

G. Koch,
Profile Decompositions and Navier-Stokes, (joint work with I. Gallagher and F. Planchon).
We use the dispersive method of "critical elements" established by Kenig and Merle to give an alternative proof of a well-known Navier-Stokes regularity criterion due to Escauriaza, Seregin and Sverak. The key tool is a decomposition into profiles of bounded sequences in critical spaces.

E. Shargorodsky,
Bernoulli free-boundary problems.
A Bernoulli free-boundary problem is one of finding domains in the plane on which a harmonic function simultaneously satisfies the homogeneous Dirichlet and a prescribed inhomogeneous Neumann boundary conditions. The boundary of such a domain is called a free boundary because it is not known a priori. The classical Stokes waves provide an important example of a Bernoulli free-boundary problem. Existence, multiplicity or uniqueness, and smoothness of free boundaries are important questions and their solutions lead to nonlinear problems.
The talk, based on a joint work with J.F. Toland, will examine an equivalence between these free-boundary problems and a class of nonlinear pseudo-differential equations for real-valued functions of one real variable, which have the gradient structure of an Euler-Lagrange equation and can be formulated in terms of the Riemann-Hilbert theory. The equivalence is global

in the sense that it involves no restriction on the amplitudes of solutions, nor on their smoothness.
Non-existence and regularity results will be described and some important unresolved questions about how irregular a Bernoulli free boundary can be will be formulated.

C.-J. Xu,
Well-posedness and qualitative properties for Boltzmann equation without angular cutoff.
It is known that the singularity in the non-cutoff cross-section of the Boltzmann collision operator leads to the gain of regularity in the velocity variable. By defining and analyzing a new non-isotropic norm which precisely captures the dissipation in the linearized collision operator, we first give a precise coercive estimate for general physical cross-sections. Then the Cauchy problem for the Boltzmann equation is considered in the framework of small perturbation of an equilibrium state where the global existence of classical solution is established in a general setting. With some essential estimates on the collision operators, the proof is based on the energy method through macro-micro decomposition.
Furthermore, we study the qualitative properties of solutions, precisely, the full regularization in all variables, uniqueness, non-negativity and convergence rate to the equilibrium. The key step to obtain the regularizing effect is a generalized version of the uncertainty principle together with a theory of pseudo-differential calculus on non-linear collision operators.
In summary, the above results lead to a satisfactory mathematical theory for the space inhomogeneous Boltzmann equation without angular cutoff. The results of this talk are from a series of joint works with R. Alexandre, Y. Morimoto, S. Ukai and T. Yang.

Session 10, October 8, 2010 in London

P. Gérard,
Action-angle variables for the cubic Szegő equation.
The cubic Szegő equation is an evolution equation on the Hardy space of the circle, which is a toy model for infinite dimensional Hamiltonian systems without dispersion. It turns out that this system admits a Lax pair. In this

talk I will show how to use this Lax pair structure to construct explicitly the action-angle variables for generic data, and discuss some applications to stability theory of special solutions. This is a joint work with Sandrine Grellier (Orléans).

M. Hairer,

Spatially Rough (S)PDEs.

We consider a class of $1+1$-dimensional Burgers-type equations driven by space-time white noise. These equations arise naturally in some path sampling problems. Their main features are that the nonlinearity is not a total derivative and that the solutions are spatially quite rough. As a consequence, the standard weak formulation breaks down and it is not clear what the right concept of solution should be. We propose a solution concept based on the theory of rough paths, which allows to understand very clearly in which sense the equations are ill-posed and how different approximations can converge to different solutions.

A. Pushnitski,

Scattering matrix and the spectral theory of discontinuous functions of self-adjoint operators.

Let A and B be self-adjoint operators in a Hilbert space such that the scattering matrix for the pair A, B is well defined. I will discuss the Fredholm index of the pair of spectral projections $P(A), P(B)$, corresponding to the interval $(-\infty, E)$. It turns out that that this index is related to the eigenvalue counting function of the scattering matrix $S(E)$ for the pair A, B. The formula which expresses this relationship can be interpreted as an integer valued version of the Birman-Krein formula.

A. Shirikyan,

Exponential stabilisation to a non-stationary solution for Navier-Stokes equations and applications.

The problem of controllability and stabilisation for Navier-Stokes equations in a bounded domain was intensively studied in the last twenty years. In particular, it was proved that the Navier-Stokes system is exactly controllable in any finite time by an external force localised in space, and any stationary point of the flow can be stabilised by a finite-dimensional feedback control. This talk is devoted to the problem of stabilisation to a non-stationary solution of Navier-Stokes equations. We show that it can be achieved by a finite-dimensional force localised in space and time. We also discuss two applications of this result: construction of a feedback control stabilising a given smooth solution and exponential mixing of the flow for $2D$ Navier-Stokes equations perturbed by a space-time localised noise. The results presented in this talk are obtained in collaboration with V. Barbu, S. Rodrigues, and L. Xu.

Session 9, May 31, 2010, in Paris

R. Danchin,
A global existence result for the compressible Navier-Stokes equations in the critical L^p framework.
This talk is dedicated to the global well-posedness issue for the barotropic compressible Navier-Stokes system in the whole space. We aim at using a "critical functional framework" which is not related to the energy space. For small perturbations of a stable equilibrium state in the sense of suitable L^p-type Besov norms, we establish global existence. As a consequence, like for incompressible flows, one may exhibit a class of large highly oscillating initial velocity fields for which global existence and uniqueness hold true. The proof is based on new estimates for the linearized and the paralinearized system. This is a joint work with F. Charve.

P. Markowich,
Bohmian measures and their classical limit,
(based on joint work with Thierry Paul and Christoph Sparber). We consider a class of phase space measures, which naturally arise in the Bohmian interpretation of quantum mechanics (when written in a Lagrangian form). We study the so-called classical limit of these Bohmian measures, in dependence on the scale of oscillations and concentrations of the sequence of wave functions under consideration. The obtained results are consequently compared to those derived via semi-classical Wigner measures. To this end, we shall also give a connection to the theory of Young measures and prove several new results on Wigner measures themselves. We believe that our analysis sheds new light on the classical limit of Bohmian quantum mechanics and gives further insight on oscillation and concentration effects of semi-classical wave functions.

G. Seregin,
How does L^3-norm approach potential blowup of the Navier-Stokes equations?.
In the talk, we are going to discuss behavior of L^3-norm approaching potential blowup of the Navier-Stokes equations. Although the full answer is still unknown, some partial results will be presented.

S. Vu Ngoc,
Symplectic and spectral theory of semitoric integrable systems.
Semitoric systems form a class of completely integrable systems with two degrees of freedom that naturally generalizes completely integrable hamiltonian torus actions. I will report on recent results obtained with Alvaro Pelayo on the symplectic classification of such systems. Then the quantum analogue of these systems will be presented, together with results and conjectures concerning their spectral theory, in the semiclassical limit.

Session 8, March 19, 2010, in London

Y. Guivarc'h,
A renewal theorem for products of random matrices and some applications to stochastic recursions.
We consider a product of i.i.d random matrices A_k, and we investigate the asymptotics of the length of colum vectors for the product, under natural hypotheses on the law of A_k. We establish a renewal theorem, extending the classical one for sums of i.i.d real random variables as well as Kesten's results for positive matrices. An important role in the corresponding analysis is played by certain spectral properties of group actions on projective spaces and associated operators. We describe some consequences:
(a) Asymptotics of the tails of stationary laws for multidimensional stochastic recursions.
(b) Properties of the solutions of a matrix stochastic equation of Mandelbrot type.
(c) Convergence towards stable laws for the sum of increments associated with multidimensional stochastic recursions.

B. Helffer,

On spectral problems related to a time dependent model in superconductivity with electric current.

This lecture is mainly inspired by a paper of Y. Almog which appeared last year in the *Siam J. Math. Anal.* Our goal here is first to discuss in detail the simplest models which we think are enlightening for understanding the role of the pseudospectra in this question and secondly to present proofs which will have some general character and will for example apply in a more physical model, for which we have obtained recently results together with Y. Almog and X. Pan.

S. Kuksin,

Perturbed KdV.

I consider perturbations of the KdV equation under periodic boundary conditions which may include a random force. For any perturbation I heuristically derive an effective equation which describes the behaviour of solutions for the perturbed equation on long time-intervals. These new equations do not contain the small parameter and are often well-posed. For some classes of perturbations with randomness I prove that, indeed, the effective equation correctly describes the long time dynamics of solutions. I will discuss the relations of these results with the classical finite-dimensional averaging as well as with the Whitham averaging.

B. Niethammer,

Self-similar rupture of thin films with slip.

We consider a simple model for line rupture of thin fluid films in which Trouton viscosity and van-der-Waals forces balance. For this model there exists a one-parameter family of second kind self-similar solutions describing the evolution towards the point of rupture. We establish necessary and sufficient conditions for convergence to those self-similar solutions that lie in a certain parameter regime and present results of numerical simulations that support a conjecture on the domains of attraction of all self-similar solutions.

Session 7, December 4, 2009, in Paris

M. Dafermos,

TBA.

D. Dos Santos Ferreira,

Stability estimates for anisotropic inverse problems.

We are interested in the following inverse problem for evolution equations: in a compact Riemannian manifold with boundary, find the potential or the conformal factor of the metric from the knowledge of the dynamical Dirichlet-to-Neumann map. For instance, for the wave equation the question of identifiability has been settled by Belishev and Kurylev using the boundary control method, introduced by Belishev. This method however doesn't seem to provide suitable stability estimates.

Following ideas of Stefanov and Uhlmann on the wave equation, and inspired by a recent paper of DSF, Kenig, Salo and Uhlmann (concerned with the anisotropic Calderón problem), we derive stability estimates in simple geometries for potentials and close conformal factors from the Dirichlet-to-Neumann map associated to the dynamical Schrödinger equation. This a joint work with Mourad Bellassoued (Faculté des Sciences de Bizerte).

H. Isozaki,

Spectral deviations for the damped wave equation.

We consider an inverse problem associated with some 2-dimensional non-compact manifolds, or strictly speaking, orbifolds. Our motivating example is a Riemann surface $\mathcal{M} = \Gamma\backslash\mathbf{H}^2$ associated with the Fuchsian group of 1st kind Γ containing parabolic elements. $\mathcal{M}$ is non-compact, and has a finite number of cusps and elliptic singular points. It is then regarded as a hyperbolic orbifold. We introduce a class of Riemannian orbifolds whose metric is asymptotically equal to that of this Riemann surface at the cusp, and by observing solutions of the Helmholtz equation at the cusp, define a generalized S-matrix. We then show that this generalized S-matrix determines the Riemannian metric.

C. Mouhot,

On Landau damping.

Landau damping is a collisionless stability result of considerable importance in plasma physics, as well as in galactic dynamics. Our recent work on the subject provides a first mathematical ground for this effect in the nonlinear regime, and qualitatively explains its robustness over extremely long time scales. This is a joint work with C. Villani.

Session 6, October 2, 2009, in London

N. Anantharaman,
Spectral deviations for the damped wave equation.
We prove a Weyl-type fractal upper bound for the spectrum of the damped wave equation, on a negatively curved compact manifold. It is known that most of the eigenvalues have an imaginary part close to the average of the damping function. We count the number of eigenvalues in a given horizontal strip deviating from this typical behaviour; the exponent that appears naturally is the "entropy" that gives the deviation rate from the Birkhoff ergodic theorem for the geodesic flow. A Weyl-type lower bound is still far from reach; but in the particular case of arithmetic surfaces, and for a strong enough damping, we can use the trace formula to prove a result going in this direction.

J. Ball,
Interfaces, surface energy and solid phase transformations.
Phase transformations in solids lead to interfaces between different variants of the product phase. In some materials these are atomistically sharp, while in others the interface thickness extends over a number of atomic spacings. The talk will discuss different variational models for describing such interfaces and their analysis. This is joint work with Elaine Crooks (Swansea) and with Carlos Mora-Corral (Bilbao).

T. Duyckaerts,
Non-generic blowup solutions to cubic focusing inhomogeneous nonlinear Schrödinger equations in two dimensions.
The nonlinear Schrödinger equations with a focusing cubic nonlinearity admits an explicit blowup solution based on the ground state of the equation, which has minimal mass among the blowup solutions and an unstable blowup behavior. In this talk I will construct similar unstable blowup solutions in

the presence of an external potential and when the nonlinearity is inhomogeneous. This construction is based on properties of the linearized operator around the ground state, and on a full use of the invariances of the homogeneous cubic equation, via time-dependent modulations. This is a joint work with Valeria Banica (Evry) and Rémi Carles (Montpellier).

J. Langley,

Zeros of derivatives in the plane and off the real line.

It follows from the classical Polya shire theorem that if the function f is meromorphic with at least two distinct poles in the plane then for all sufficiently large k the kth derivative $f^{(k)}$ has at least one zero. It was conjectured by Gol'dberg that the frequency of distinct poles of f is in fact controlled by the frequency of zeros of $f^{(k)}$ as soon as $k \geq 2$. This is known to be true if all poles of f have multiplicity at most $k-1$ (Frank-Weissenborn). It is also known that if two derivatives $f^{(m)}$ and $f^{(n)}$ have finitely many zeros, where $0 \leq m \leq n-2$, then f has finitely many poles (Frank, JKL). A recent result shows that if f grows not too fast, and if $f^{(k)}$ has finitely many zeros for some $k \geq 2$, then again f has finitely many poles. On the other hand, simple examples make it clear that no result along the lines of Gol'dberg's conjecture holds for $k = 1$.

The second main theme concerns non-real zeros of derivatives of real entire functions and results by several authors (Levin, Ostrovskii, Hellerstein, Williamson, Sheil-Small, Edwards, Bergweiler, Eremenko, JKL) arising from a conjecture of Wiman around 1911. Let f be an entire function, real on the real axis, with only real zeros. If $f^{(k)}$ has only real zeros for some $k \geq 2$, then f belongs to the Laguerre-Polya class LP of locally uniform limits of real polynomials with real zeros, and all derivatives of f have only real zeros. On the other hand if f does not belong to LP then the number of non-real zeros of $f^{(k)}$ tends to infinity with k.

The last part of the talk involves non-real zeros of the derivatives of meromorphic functions. Here much less is known than in the entire case, but some recent results suggest that at least some progress is possible.

Session 5, June 4, 2009, in Paris

N. Bournaveas,
Kinetic models of chemotaxis.
Chemotaxis is the directed motion of cells towards higher concentrations of chemoattractants. At the microscopic level it is modeled by a nonlinear kinetic transport equation with a quadratic nonlinearity. We'll discuss global existence results obtained using dispersion and Strichartz estimates, as well as some blow up results (joint work with Vincent Calvez, Susana Gutierrez and Benoît Perthame).

D. Lannes,
The stabilizing role of gravity for Rayleigh-Taylor instabilities in internal waves.
The internal waves problem consists in studying the motion of the interface between two perfect fluids of different densities (the water waves problem is thus the particular case of an upper fluid of density zero). Even if the heavier fluid is below, it is known that in absence of surface tension, these equations are always ill-posed in absence of surface tension, due to the formation of Rayleigh-Taylor/Kelvin-Helmoltz instabilities. However, these results do not "fit" with in situ observations that show the propagation of very large internal waves over very long distances in situations where the surface tension is very small. By a careful analysis of the role played by gravity, we are able to give a lower bound for the appearance of such instabilities, and thus get a mathematical result closer to the physical observations.

T. Paul,
Quantum normal forms and long time semiclassical approximation.
A "non-symbolic" operator theoretic derivation of the quantum Birkhoff canonical form near a periodic trajectory is presented, and provides an explicit recipe for expressing this "quantum" form from the usual (symbolic) construction (work in collaboration with Victor Guillemin). This construction can be used in the study of the long time (diverging as the Planck constant vanishes) semiclassical evolution, for which several results are presented, with an emphasis on the cases where the classical paradigm is not recovered at the (semi)classical limit.

N. Tzvetkov,
Transverse instability of the water solitary waves.

We present a recent joint work with F. Rousset. We prove the instability of the water line solitary waves constructed by Amick-Kirchgassner with respect to transverse perturbations. For that purpose we construct a family of solutions of the water waves equations which are initially arbitrary close to the line solitary waves, but for later (long) times they separate from the solitary wave (and its spatial translates) at some fixed distance, the distance being measured in some natural norms for the considered problem.

Session 4, March 20, 2009, in London

K. Ball,

From Monotone Transportation to Probability.

The talk will explain an elementary construction of the monotone transportation map of Brenier and how the map can be used to prove a deviation inequality of Marton and Talagrand and to understand entropy growth for sums of random variables.

F. Germinet,

Poisson Statistics for Eigenvalues of Continuum Random Schrödinger Operators.

We show the absence of energy levels repulsion for the eigenvalues of random Schrödinger operators in the continuum. We prove that, in the localization region at the bottom of the spectrum, the properly rescaled eigenvalues of a continuum Anderson Hamiltonian are distributed as a Poisson point process with intensity measure given by the density of states. We also obtain simplicity of the eigenvalues. We derive a Minami estimate for continuum Anderson Hamiltonians. We also give a simple and transparent proof of Minami's estimate for the (discrete) Anderson model. (Joint work with Abel Klein and Jean-Michel Combes)

J. Keating,

Quantum chaotic resonance eigenfunctions.

I will review some rather interesting conjectures concerning the distribution of resonances in quantum chaotic scattering systems. I will then describe some recent results concerning the morphology of the associated resonance eigenfunctions for a particular class of open maps.

M. Lewin,

Spectral pollution and how to avoid it.

Spectral pollution is a very common phenomenon occurring when the spectrum of a self-adjoint operator is approximated using increasing finite-dimensional subspaces. Spurious eigenvalues can appear in gaps of its essential spectrum. In this work we precisely localize the spurious spectrum when some constraints are imposed on the basis (for instance if it is chosen in accordance with a decomposition of the underlying Hilbert space into a direct sum of two subspaces). This is followed by applications to Dirac and periodic Schrödinger operators. Joint work with Eric Séré (Paris Dauphine).

Session 3, December 1rst, 2008, in Paris

C. Guillarmou,

Analysis of the bottom of the spectrum for Schrödinger operators and some applications to Riesz transforms.

We will explain how to study the resolvent (or the spectral measure) near 0 for short range Schrödinger type operators on asymptotically Euclidean metric (or more generally asymptotically conic). Then we shall give some applications to boundedness of Riesz transforms on L^p and other problems coming from harmonic analysis.

K. Pravda-Starov,

Spectra and semigroup smoothing for non-elliptic quadratic operators.

We study non-elliptic quadratic differential operators. Quadratic differential operators are non-selfadjoint operators defined in the Weyl quantization by complex-valued quadratic symbols. When the real part of their Weyl symbols is a non-positive quadratic form, we point out the existence of a particular linear subspace in the phase space intrinsically associated to their Weyl symbols, called a singular space, such that when the singular space has a symplectic structure, the associated heat semigroup is smoothing in every direction of its symplectic orthogonal space. When the Weyl symbol of such an operator is elliptic on the singular space, this space is always symplectic and we prove that the spectrum of the operator is discrete and can be described as in the case of global ellipticity. We also describe the large time behavior of contraction semigroups generated by these operators.

H.H. Rugh,

Cones and gauges in complex spaces.

We introduce the notion of a complex cone and its associated projective gauge. This generalises a real cone with its Hilbert metric (introduced by Birkhoff in 1957). We prove various spectral gap theorems for operators contracting a complex cone. In particular we prove a Perron-Frobenius like Theorem for a certain class of complex matrices.

J. Wright,

Isoperimetric(-type) inequalities in harmonic analysis.

Many inequalities in harmonic analysis can be viewed in the same framework as the Loomis-Whitney inequality (which immediately implies the classical isoperimetric inequality). In this talk we will develop this point of view.

Session 2, October 3rd, 2008, in London

E.B. Davies,

Spectrum of a rotating fluid film.

A few years ago some non-rigorous and partly numerical calculations suggested that a certain highly non-self-adjoint and non-elliptic second order ordinary differential operator arising in fluid mechanics had real spectrum. In this lecture I will explain results of John Weir and myself that eventually proved that this was indeed the case. The implications for the original fluid evolution equation are spelt out.

J. Dolbeault,

Fast diffusions and generalized entropies.

The first part of the talk corresponds to a joint work with B. Nazaret and G. Savaré. It will be devoted to inequalities which interpolate between logarithmic Sobolev and Poincaré inequalities and give rates for equations involving an Ornstein-Uhlenbeck operator. The method is based on a non-local Bakry- Emery criterion and can be adapted to nonlinear diffusions of porous medium type. The second part of the talk is concerned with intermediate asymptotics of nonlinear diffusions (porous medium and fast diffusion). It is based on two papers written with A. Blanchet, M. Bonforte, G. Grillo and J.L. Vázquez. In self-similar variables, by linearizing the entropy and entropy-production functionals, it is shown that getting optimal rates amounts to find optimal constants in some Hardy-Poincaré inequalities.

I. Gallagher,
Spectral asymptotics for a skew-symmetric perturbation of the harmonic oscillator.
The aim of this lecture is to discuss spectral and pseudospectral properties of a perturbed harmonic oscillator. Motivated by a problem in Fluid Mechanics, we are particularly interested in the asymptotics of the infimum of the real part of the spectrum, as the parameter goes to zero. This is a joint work with Thierry Gallay and Francis Nier.

M. Ruzhansky,
Pseudo-differential operators and symmetries.
The theory of pseudo-differential operators on manifolds usually relies on local representations of operators in local coordinates, thus often ignoring global geometric and algebraic information that is often available. We will present a new approach to pseudo-differential operators exploring global symmetries of the underlying space and yielding a globally defined full symbol. The talk will be based on the joint work with Ville Turunen (Helsinki).

First session of the Paris London Analysis Seminar, May 16, 2008, in Paris

R. Carles,
Loss of regularity for supercritical nonlinear Schrödinger equations.
We consider the nonlinear Schrödinger equation with defocusing, smooth,

nonlinearity. Below the critical Sobolev regularity, it is known that the Cauchy problem is ill-posed. We show that this is even worse, namely that there is a loss of regularity, in the spirit of the result due to G. Lebeau in the case of the wave equation. As a consequence, the Cauchy problem for energy-supercritical equations is not well-posed in the sense of Hadamard. We reduce the problem to a supercritical WKB analysis. The proof of the main result relies on the introduction of a modulated energy functional à la Brenier. This is a joint work with T. Alazard.

A. Its,

Global asymptotic analysis of the Painlevé transcendents.

In this talk we will review some of the global asymptotic results obtained during the last two decades in the theory of the classical Painlevé equations with the help of the Isomonodromy - Riemann-Hilbert method. The results include the explicit derivation of the asymptotic connection formulæ, the explicit description of linear and nonlinear Stokes phenomenon and the explicit evaluation of the distribution of poles. We will also discuss some of the most recent results emerging due to the appearance of Painlevé equations in random matrix theory. The Riemann-Hilbert method will be outlined as well.

S. Olla,

Diffusion (and superdiffusion) of energy in a system of oscillators.

We consider a system of coupled oscillators whose Hamiltonian dynamics is perturbed by stochastic terms that conserve energy (and eventually momentum). We study the macroscopic thermal conductivity and the diffusion of energy in the kinetic limit. The corresponding Wigner distribution of the energy converges to a linear phonon Boltzmann equation. In the one dimensional unpinned case (with conservation of momentum and energy) this Boltzmann equation generates a superdiffusion governed by a Levy process, i.e. a fractional heat equation governs the macroscopic evolution of the energy.

L. Parnovski,

Spectral properties of periodic pseudo-differential operators.

I will discuss some recent results in the spectral theory of self-adjoint periodic elliptic problems (joint papers with G.Barbatis, R.Shterenberg and

A.Sobolev). One type of results I will present is the proof of the Bethe-Sommerfeld conjecture (the finiteness of the number of spectral gaps) for a large class of pseudo-differential operators which include, in particular, magnetic Schrödinger operators. The conditions under which the conjecture is shown to hold are very close to the optimal ones. The second result is a complete asymptotic expansion for the integrated density of states of a two-dimensional electric Schrödinger operator.